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FOR PUBLIC AND PRIVATE SCHOOLS.

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1872.

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NATURAL SERIES.

THE

ANALYSIS

OF

INTELLECTUAL ARITHMETIC,

WITH

Mental and Blackboard Exercises.

DESIGNED FOR

PUBLIC AND PRIVATE SCHOOLS.

BY

S. A. FELTER, A. M.,

LATE OF THE BROOKLYN COLLEGIATE AND POLYTECHNIC INSTITUTE, AUTHOR OF A SERIES OF SCHOOL ARITHMETICS, ETC.

NEW YORK:
PUBLISHED BY CHARLES SCRIBNER & CO.
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PREFACE.

The principal object of this book is to assist the teacher in leading the pupil gradually and systematically from the analysis of the simple to that of the complex problem, a peculiarity of this series which has received universal commendation.

The distaste for the study of intellectual arithmetic, which is frequently found in schools, is chiefly due to the inability on the part of pupils to perform, readily, mental calculations. To make them prompt, quick, and practical, numerous blackboard exercises and brief methods of combining numbers in the fundamental rules, fractions, and interest, are introduced.

To more clearly unfold by mental exercises the principles of arithmetic, it is thought that the same formulas should be used both in mental and written, the same methods of analysis given, similar slate and blackboard exercises introduced, and the subjects presented in the same order, that the pupil may fully *appreciate* that mental and written arithmetic are one and the same thing in fact and expression.

The exercises for general analysis are so arranged, that the simple precedes and illustrates the complex, and so comprehensive as to embrace every variety of language by which mathematical relations are expressed in problems. This is a new feature, which it is hoped, will give a stimulant to healthy mental activity.

The author has endeavored to give a practical exposition of the Metric system of weights and measures. By retaining the names of such only of the denominations as are necessary in the transaction of business, he has simplified the cumbrous nomenclature which forms so great an obstacle to its general introduction.

In view of the cordial reception which has been accorded to the former numbers of the *Natural Series* by his fellow teachers and the friends of education generally, the author cannot forbear to hope that this effort may meet with a like kind and sympathetic response; and he, at the same time, would tender his sincere thanks for the assistance rendered by valuable suggestions and timely criticisms.

New York, Jan., 1868.

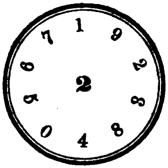
INTELLECTUAL ARITHMETIC.

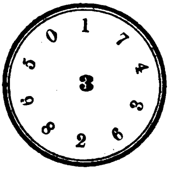
ADDITION.

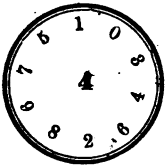
LESSON I.

BLACKBOARD EXERCISES.

Model Operations.







NOTE.—The above black-board exercises are to be recited as follows:—

Beginning at one, go round the circle five times, to the right and to the left alternately, naming the sum of the figure at the centre and each successive figure on the circumference.

90	80	70	60	50	40	30	20	10
30	70	90	50	60	10	20	40	80
—	—	—	—	—	—	—	—	—

FORMULA.—3 tens and 9 tens are 12 tens; 12 tens are equal *to 120 units.*

1. **Rule.**[*]—I. *Annex a cipher to the sum of the left hand column; thus:* 12, 120; 15, 150; 16, 160; 11, 110; *&c.*

SUGGESTIONS TO THE TEACHER.

Preparation.[†]—Each pupil should write on his slate *ten* exercises like the model, and repeat mentally the sum of each example, (See Rule,) at least five times, commencing at the left and the right alternately.

Recitation.—*Exercise* 1. Let the Teacher write on the blackboard *ten* exercises like the model; and the pupil should recite them in the same manner. (See Rule.)

Ex. 2. Require the pupils, singly or together, to give the sum of each example as soon as it is pointed to by the Teacher; thus: 120, 150, 160, &c.

Ex. 3. Require the pupils, singly or in concert, to give the sum instantly without the use of either slate or of blackboard.

N. B.—Pupils should be required to commit THOROUGHLY the above and each of the following blackboard exercises before proceeding.

PROBLEMS.

Containing but one Elementary Question.

Mary has 30 books, and Margaret has 40; how many have both?

Arithmetical Formula.—*If Mary has* 30 *books and Margaret has* 40, *both have the sum of these numbers, which is* 70 *books; hence both have* 70 *books.*

---

[*] Note.—The following rules for recitation are but brief directions, intended to aid in quick and accurate addition. The formulas need not be used after they are understood.

[†] Note.—The following notes in regard to recitation and study are intended to be suggestive only. The teacher should vary both according to circumstances.

1. If Susan has fifty apples and Jane 90; how many have both?

2. Eliza is 30 years old and Laura is 60; what is the sum of their ages?

3. George placed 40 nuts in one pile and 90 in another; how many nuts in both piles?

4. A man bought a cow for $30, and a horse for $90; what was the cost of both?

5. A lady paid 80 cents for silk, and 40 cents for ribbon; how much did she pay for both?

6. Charles paid 30 cents for a slate and 80 cents for a book; how much did he pay for both?

7. Hannah counted 30 apples in one basket, and 50 in another; how many apples in both?

8. John is 40 years old, and Susan is 20 years older; how old is Susan?

9. Little Peter is 10 years old, and his father is 30 years older; how old is his father?

10. Robert is 30 years older than John, and John is 20; how old is Robert?

Suggestions.—In addition to the above, each pupil may hand to the teacher in writing, one or more problems like the above, to be read to the class for solution. This will do much to break the *routine* into which classes in mental arithmetic are very apt to fall.

EXERCISES IN ADDITION.

What are the sums of the following series of numbers?

1.—3, 2, 4, 1, 4, 2, 4, 3, 2, 4, 3, 2, 0, 1, 2, 3, 4, 2, &c.
2.—3, 1, 3, 2, 4, 1, 3, 2, 4, 1, 2, 3, 3, 1, 4, 2, 1, 4, &c.
3.—4, 1, 2, 3, 2, 4, 3, 2, 1, 3, 4, 2, 2, 1, 3, 3, 4, 2, &c.
4.—4, 2, 5, 3, 3, 2, 1, 1, 4, 2, 3, 0, 5, 3, 4, 4, 1, 2, &c.
5.—5, 2, 3, 4, 2, 3, 4, 1, 5, 4, 4, 2, 0, 3, 2, 1, 4, 3, &c.

STUDY.—The pupil should add mentally as follows: 3, 5, 9, 10, 14, 18, &c., alternating backward and forward, until able to obtain the same result *five* times in succession.

RECITATION.—*Ex.* 1. Require each pupil in the class to read aloud one or all of these exercises in the same manner as they are to be studied.

Ex. 2. Require the pupils to add mentally, while the teacher dictates the numbers. Each scholar, after he has completed the addition, may raise his right hand. Let the teacher then call upon some pupil to give a result. Those who have not the same answer, should keep their hands up, until a result like their own is given, &c. Finally let the teacher give the correct answer.

LESSON II.

BLACKBOARD EXERCISES.

MODEL OPERATIONS.

RECITATION.—(See Note to Lesson I.)

87	35	40	58	90	67	80	94
60	30	79	20	87	60	56	30

14, 7; 147.

FORM *—80 and 60 are 140; 140 and 7 are 147.

2. Rule.—II. *To the sum of the tens annex the unit figure: thus;* 14, 7, 147; 11, 4, 114; 6, 5, 65; &c.

STUDY AND RECITATION.—(See Notes to Lesson I.)

PROBLEMS.

1. Ellen has 80 cherries on one tree, and 36 on another; how many cherries on both?

FORM.—(See Lesson I.)

---

*N. B.**—*The formulas* should be omitted as soon as the operation is understood.

2. George bought 5 books for 90 cents, and a slate for 47 cents; how much did both cost him?

3. If you give 80 cents for a pair of gloves, and 86 cents for a neck-tie, how much will both cost?

4. A bookseller sold 5 arithmetics for 99 cents, and 2 readers for 50 cents; how much did he receive for both?

5. For 7 sheep a farmer received $70, and for a cow $68; how much did he get for both?

6. A farmer sold 60 sheep, and kept 49; how many had he at first?

7. A farmer sold 80 bushels of oats, and had 85 left; how many bushels had he at first?

8. Charles is 43 years older than James; James is 20 years old; how old is Charles?

9. John was 20 years older than Richard; Richard died when he was 55 years old; how old was John at that time?

10. A boy spent 90 cents, and had 88 cents left; how many cents had he at first?

EXERCISES IN ADDITION.

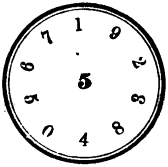
What are the sums of the following series of numbers?

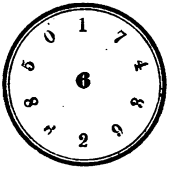
1.—3, 4, 5, 6, 7, 1, 3, 4, 2, 5, 7, 1, 3, 4, 7, 5, &c.
2.—2, 4, 6, 7, 5, 3, 2, 6, 5, 4, 2, 7, 1, 3, 6, 5, &c.
3.—7, 5, 6, 4, 3, 1, 2, 3, 2, 4, 7, 3, 5, 4, 1, 3, &c.
4.—5, 6, 5, 7, 5, 2, 3, 2, 1, 2, 3, 4, 7, 5, 6, 5, &c.
5.—4, 3, 7, 6, 4, 2, 1, 4, 4, 7, 6, 3, 5, 4, 1, 2, &c.

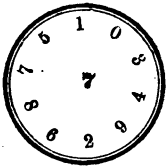
STUDY AND RECITATION.—(See Lesson I. Note.)

LESSON III.

BLACKBOARD EXERCISES.







RECITATION.—(See Lesson I. Note.)

63	82	45	58	63	72	85	91
72	73	74	71	76	77	73	78

13, 5; 135.

FORM.—70 and 60 are 130; 2 and 3 are 5; 130 and 5 are 135.

3. **Rule.**—III. *To the sum of the tens annex the sum of the units: thus;* 13, 5, 135; 10, 8, 108; 15, 5, 155, &c.

PROBLEMS.

1. If I pay $19 for a saddle, and $30 for a horse; how much do I pay for both?

2. Andrew paid 36 cents for 8 quarts of nuts, and 83 cents for a loaf cake; how much did both cost?

3. James sold 8 melons for 93 cents, and his brother Henry sold 11 melons for 25 cents more; *how much did Henry* receive for melons?

4. Nine pens are worth 11 cents, and 20 are worth 56 cents, how much are all worth?

5. A farmer sold 2 pounds of butter for 93 cents, and 3 pounds for 94 cents; how much did he get?

6. A man bought 25 pounds of cheese for $2 at one time; and at another time 83 pounds for $12; how many pounds of cheese did he buy?

7. A miller sold 27 bushels of corn, and had 42 bushels left; how many bushels had he at first?

8. Bought a carriage for 60 dollars, and a harness for 35 dollars; how much did both cost?

EXERCISES IN ADDITION.

(1.)	(2.)	(3.)	(4.)
26+30= 56	41+67=	46+32	34+82
41+67=108	34+45=	71+63	67+83
34+41= 75	23+86=	81+87	94+62
67+81=148	39+40=	42+97	33+86
93+41=134	37+62=	46+33	41+72
41+72=113	41+68=	96+43	41+67

Preparation and Recitation.—*Exercise.*—1. The above, and the following exercises are to be copied and completed on slates or paper at the seats, (see Ex. 1,) and then read with the answers at recitation.

Ex. 2. Let the class complete the exercises from the book, giving the answers aloud.

Ex. 3. Let the class be thoroughly questioned, and let the answers be given without the book.

Ex. 4. Should the above and the following exercises be insufficient to make the pupils skilful in combining numbers mentally, or at sight, let the teacher dictate combinations, to be completed at the seats and recited in the usual manner.

LESSON IV.

BLACKBOARD EXERCISES.

In which the sums of the columns of units exceed nine.

38	73	34	35	36	37	38	39
77	97	87	47	76	78	87	67
10, 15; 115.							

FORM.—70 and 30 are 100; 8 and 7 are 15; 100 and 15 are 115.

4. **Rule.**—IV. *Write the sums of the tens and units separately; add the tens of each sum to the units of the preceding sum; thus:* 10, 15; 1 (0+1) 5; 115: 10, 11; 1 (0+1) 1; 111: 16, 10; 1 (6+1) 0; 170, &c.

STUDY AND RECITATION.—(See Lesson I. Note.)

PROBLEMS.

1. Charles has two notes due him, one for $48, and the other for $78; how much is due him in all?

2. If I pay $57 for sugar, and $27 for salt; how much do I pay for both?

3. John gave 20 cents for 38 sheets of paper, and 94 cents for a slate; how much did both cost?

4. A merchant paid $37 for some sugar, and $46 for some molasses; how much did both cost?

5. There are 94 pens in one box, and 86 pens in *another; how* many have both?

6. A man rode 68 miles in the cars on one day, and 59 miles the next; how many miles did he travel in all?

7. Jane has 13 pins in her cushion, and Mary has 28 in hers; how many pins in both cushions?

8. There are 2 bags of coffee in the closet, the first contains 26 pounds, the second, 18 pounds; how many pounds in both bags?

EXERCISES.

(1.)	(2.)	(3.)	(4.)
42+36=	91+37	64+87	89+67
41+37=	41+86	38+64	38+86
86+43=	98+86	93+87	97+95
84+16=	41+83	44+87	67+83
37+42=	72+46	39+64	67+37
86+47=	87+89	59+78	41+67

Study and Recitation.—See Lesson III.

LESSON V.

BLACKBOARD EXERCISES.

In which the sum of the tens does not exceed nine.

MODEL OPERATION.

828	861	939	581	631	701	838	931
467	622	422	715	467	973	865	647
——	——	——	——	——	——	——	——

12, 8, 15; 1295.

Form.—400 and 800 are 1200; 67 and 28 are 95; 1200 and 95 are 1295.

Illustration.—12, 8, 15; 1295: 10, 6, 13; 1073: *14, 8, 3; 1483.*

413	349	416	841	806	836	413
672	412	672	747	433	717	572

PROBLEMS.

1. A man rode 45 miles on one day, and 67 miles on another; how many miles did he ride on both days?

2. A man sold 80 pounds of butter at one time, and 68 pounds at another time; how many pounds did he sell in all?

3. A man paid 78 dollars for 9 calves, and 38 dollars for 20 sheep; how much did both cost?

4. Henry had 38 cents after he had given away 46; how many cents had he at first?

5. Andrew gave away 27 cents, and had 46 remaining; how many cents had he at first?

6. 20 oranges cost 76 cents; 40 apples cost 87 cents; how much did both cost?

7. A farmer gave $109 for some sheep, and $56 for a cow; how much did he give for both?

8. A lady paid $220 for a carpet, and $620 for other furniture; how much did she pay out in all?

LESSON VI.

BLACKBOARD EXERCISES.

In which the sum of the tens exceeds nine.

896	878	678	917	843	987
474	434	483	627	496	382

12, 16, 10; 1370.

Form.—400+800=1200; 74+96=170; 1200+170=1370.

Rule.—*Write the sums of the hundreds, tens, and units separately; add the tens of each sum to the units of the preceding sum; thus:* 12, 16, 10; 1 (2+1) (6+1) 0; 1370.

PROBLEMS.

1. How many are 10+1? 30+2? 50+3? 70+7? 90+4? 20+6? 40+1? 60+9? 80+8? 100+2?

2. How many are 21+2? 41+2? 51+2? 61+7? 81+9? 91+7? 11+9? 31+7? 71+5? 101+6?

3. How many are 22+6? 32+4? 12+5? 42+3? 52+8? 62+3? 82+7? 72+5? 102+8? 92+7?

4. How many are 13+5? 23+7? 33+6? 53+8? 43+4? 63+5? 73+4? 83+8? 103+7? 93+6?

5. How many are 24+8? 64+5? 84+12? 14+8? 54+6? 74+8? 94+6? 34+3? 104+7? 24+6?

6. How many are 35+4? 65+3? 85+7? 95+10? 25+6? 85+4? 75+9? 45+3? 105+9? 115+6?

7. How many are 16+3? 36+9? 46+5? 76+8? 86+4? 26+7? 96+10? 106+9? 116+8? 16+5?

8. How many are 27+8? 37+6? 47+4? 97+3? 107+6? 127+10? 17+8? 57+9? 67+7? 77+3?

9. How many are 28+8? 58+9? 68+6? 78+7? 98+9? 48+4? 38+3? 128+5? 68+7? 88+8?

10. How many are 19+6? 29+5? 39+4? 59+7? 99+9? 89+3? 49+8? 59+8? 39+3?

11. How many are 6, 8, and 3? 4, 8, and 9? 3, 7, and 8? 5, 4, and 7? 9, 5, and 4? 6, 4, and 7? 4, 3, and 10? 7, 8, and 2? 6, 5, and 4? 2, 8, and 9?

12. How many are 3, 7, and 12? 5, 9, and 7? 6, 4, and 5? 12, 2, and 7? 4, 10, and 6? 8, 4, and 6? 2, 8, and 4? 5, 8, and 11? 6, 9, and 3? 4, 2, and 7?

13. How many are 5, 8, and 7? 6, 2, and 9? 3, 4, and 10? 4, 7, and 8? 9, 6, and 3? 12, 3, and 5? 7, 6, and 2? 4, 6, and 10? 5, 8, and 6? 9, 2, and 9?

14. How many are 6, 3, and 10? 5, 8, and 7? 3, 12, and 9? 4, 10, and 8? 9, 6, and 2? 4, 3, and 8? 5, 7, and 8? 6, 8, and 9? 5, 4, and 7? 8, 10 and 3?

15. How many are 9, 11 and 2? 6, 9, and 3? 4, 3, and 7? 6, 8, and 7? 8, 4, and 11? 14, 3, and 6? 15, 2, and 9? 12, 4, and 8? 10, 3, and 7? 4, 6, and 5?

16. How many are 3, 4, and 16? 4, 6, and 5? 6, 17, and 6? 4, 7, and 12? 4, 7, and 14? 9, 11, and 16?

17. A man bought 6 apples at one time, 14 at another time, and 11 at another time; how many did he buy in all?

LESSON VII.

BLACKBOARD EXERCISES.

Containing the combination of more than two numbers.

MODEL OPERATION.

27	54	27	56	41	86	67	68	43
36	67	86	78	67	43	83	34	26
46	83	43	37	87	78	74	67	84
—	—	—	—	—	—	—	—	—

FORM.—46+30=76; 76+6=82; 82+20=102; 102+7=109.

5. Rule.—V. *To the first number add the tens and units of each number consecutively; thus:* 46, 76, 82, 102, 109; 83, 143, 150, 200, 204, *&c.*

PROBLEMS.

1. James gave 118 cents for a squirrel, 282 cents for a cage, and 215 cents for nuts; how much did he pay in all?

2. An orchard contains 15 cherry trees, 37 plum trees, and 45 apple trees; how many trees are in the orchard?

3. James is 19 years old, John 20, and Charles 22; what is the sum of their ages?

4. A farmer bought a horse for 65 dollars, a saddle for 16 dollars, and a bridle for 9 dollars; what did he pay for all?

5. A lady bought a piece of tape for 10 cents, a

paper of pins for 18 cents, and a pair of scissors for 30 cents; how much did she pay in all?

6. A farmer has 15 sheep in one field, 25 in another, and 37 in his barn-yard; how many has he in all?

7. A merchant buys 37 barrels of flour from one miller, 30 from another, and 27 from another; how many barrels does he buy in all?

8. A farmer had a horse, for which he paid 36 dollars, he sold it for 27 dollars more than he paid for it; what did he receive for it?

9. How many are 7+16? 7+26? 7+46? 7+36? 7+56? 7+96? 7+86? 7+76? 7+66?

10. How many are 8+17? 8+37? 8+27? 8+57? 8+47? 8+67? 8+97? 8+87? 8+77?

11. How many are 9+18? 9+38? 9+28? 9+58? 9+48? 9+68? 9+98? 9+78? 9+88?

12. How many are 11+19? 11+29? 11+39? 11+59? 11+49? 11+99? 11+89? 11+69?

13. How many are 12+29? 12+19? 22+49? 32+69? 42+39? 62+89? 72+79? 92+69?

14. How many are 13+69? 14+51? 26+62? 36+23? 83+31? 73+14? 43+33? 93+46? 63+73? 83+83?

15. How many are 78+62? 47+63? 89+76? 34+67? 89+37? 68+78? 34+67? 25+67? 89+67? 34+68?

16. How many are 46+37? 67+83? 60+32? 68+43? 76+86? 96+67? 34+67? 86+72? 17+62? 34+67?

17. How many are 34+67? 67+82? 46+32? 21+67? 41+67? 78+36? 68+73? 46+86? 34+66? 16+46?

18. How many are 43+72? 67+83? 43+72? 67+83? 86+83? 67+86? 34+86? 73+86? 96+27? 34+67?

(19.)	(20.)	(21.)	(22.)
64+43	87+67	413+672	913+467
87+67	43+89	837+867	389+413
72+68	97+67	413+912	671+837
17+68	38+89	671+837	419+672
16+83	41+67	413+641	418+768
91+67	49+83	916+678	491+372

SUBTRACTION.

LESSON VIII.

BLACKBOARD EXERCISES.

Model Operation.

130	130	140	70	60	90	80	110	100
60	50	80	30	60	40	70	30	80

FORMULA.—6 tens from 13 tens leave 7 tens; 7 tens are equal to 70 units.

8. **Rule.**—I. *Annex a cipher to the difference of the tens; thus:* 7, 70; 8, 80; 6, 60; &c.

Study.—The pupil should write on his slate ten exercises like the model, and mentally repeat the difference of each example (see Rule) at least five times, commencing at the right and left alternately.

Recitation.—*Exercise* 1. Let the Teacher write on the blackboard *ten* exercises like the model, and the pupils recite (see Rule) in the usual manner.

Ex. 2. Require the pupils, singly or in concert, to give the difference of each example instantly, when pointed to by the teacher; thus: 70, 80, 60, &c.

Ex. 3. Require the pupils, singly or in concert, to give the difference instantly without using either slate or blackboard.

Ex. 4. Let the Teacher question the class thoroughly.

PROBLEMS

Containing but one Elementary Question.

1. Charles had 60 apples, and he gave 40 to Henry; how many had he left?

Arithmetical Formula.—*If Charles had* 60 *apples, and he gave* 40 *to Henry, he had as many apples left as the difference between the numbers, which is* 20; *hence he had* 20 *apples left.*

2. Henry had 50 nuts, and he gave them all away; how many had he left.

3. Harvey had 80 cents, and he gave 30 to his sister; how many had he left?

4. Susan solved 80 problems in one week, and 20 problems the next; how many more problems did she solve in the first week than in the second?

5. Martha had 70 cents, and she paid 20 cents for a slate; how many cents had she left?

6. James is 80 years old; and his son is 30; how *much older is* James than his son.

7. One bag contains 80 pounds of wheat, another contains 60 pounds; how much more does one bag contain than the other?

8. There were 90 peach trees in an orchard; 60 of them were blown down in a storm; how many were left standing?

LESSON IX.

EXERCISE IN MENTAL SUBTRACTION.

1. How much will be left of 100 after subtracting each of the following numbers: 2, 3, 4, 6, 0, 2, 4, 3, 2, 4, 2, 3, 4, 5, 1, 4, 2, 3, 4, 2, 1, 4?

2. How many are 100 less 2, 3, 4, 2, 5, 2, 3, 4, 2, 3, 2, 2, 4, 5, 3, 4, 2, 4, 3, 2, 4, 2, 3, 2, 4, 2, 4, 3?

3. 100 less 3, 5, 2, 2, 1, 3, 2, 5, 2, 4, 2, 3, 4, 2, 3, 2, 4, 3?

4. 100 less 4, 3, 2, 4, 5, 2, 4, 2, 3, 5, 2, 5, 2, 2, 3, 2, 4, 2, 2, 3?

5. 100 less 3, 4, 2, 3, 4, 2, 2, 4, 3, 5, 4, 2, 3, 4, 2, 3, 5, 2, 3?

STUDY.—The pupil should repeat the remainder after each consecutive subtraction; thus: (Exercise 1st) 100, 98, 95, 91, 85, &c., at least five times, each time obtaining the same result.

RECITATION.—*Exercise* 1. Let each member of the class read from the book one or all of these exercises in the same manner as they are studied. *Accuracy* and *rapidity* should be required.

Ex. 2. Let the Teacher require the class to make the subtraction

of each consecutive number mentally, while he dictates it. After each pupil has done this, let him raise his right hand, then call on some one of the class to give the answer. Those who have the same answer, let their hands fall; when the hands are all down, the Teacher may give the correct answer.

LESSON X.

BLACKBOARD EXERCISES.

In which the unit figure of the subtrahend is less than that of the minuend.

MODEL OPERATION.

121	234	546	472	781	697	987	869
30	72	41	10	90	54	80	20
9, 1; 91							

FORM.—30 from 120 are 90; 90 and 1 are 91.

Rule.—II. *To the difference of the tens, annex the difference of the unit figures; thus:* 9, 1; 91: 30, 6; 306, *&c.*

STUDY AND RECITATION.—(See Lesson IX.)

PROBLEMS.

1. There are 109 scholars in a school, and 81 are girls; how many boys are there?

2. A farm is worth $896, and a house is worth $94; how much more is the farm worth than the house?

3. A boy had 348 cents, and he gave away 84 *of them; how* many had he left?

4. A has 60 sheep, worth $526; ten of them worth $81, died; what was the value of the rest?

5. A man traveled 890 miles; on his return, he traveled the first day 60 miles; how far had he yet to travel?

6. A drove of cattle is worth $940; a cow worth $50 is taken from it; what is the value of the rest?

7. A drove of cattle is worth $864; an ox worth $72 is taken from it; what is the value of the rest?

8. A horse is worth $209; the wagon is worth $84; how much more is the horse worth than the wagon?

MENTAL EXERCISES.

Study and Recitation.—See Lesson III, Note.

LESSON XI.

BLACKBOARD EXERCISES.

In which the unit figure of the subtrahend exceeds the unit figure of the minuend.

MODEL OPERATION.

137	126	130	142	154	103	170	181	194
39	39	31	36	37	34	31	34	36
10, 8; 98								

Form.—9 from 17 leaves 8; 30 from 120 leave 90; 8 and 90 are 98.

7. **Rule.**—III. *Diminish the difference of the tens by one, and annex the unit figure: thus;* 10, 8; 98: 15, 8; 148: 9, 7; 87: *&c.*

PROBLEMS.

1. Mary went shopping with 120 cents in her purse, and when she returned she had only 86 cents left; how much did she spend?

2. I sold a lot of wood for $180, and received in payment, cloth worth $79; how much was still due?

3. A man paid $130 to the merchant, and $60 to the blacksmith; how much more did he pay to one than the other?

4. A man traveled 194 miles on a journey; while returning the first day he traveled 55 miles; how many miles had he yet to travel?

5. A cistern which holds 110 gallons, was full in the morning, but at night there were but 31 gallons; how many gallons had leaked out?

6. A farmer bought a carriage for $120, and a harness for $62; how much more did the carriage cost than the harness?

7. Two barrels of potatoes weigh 190 pounds; one of the barrels weighs 87 pounds; what does the other weigh?

8. Two bags of grain weigh 135 pounds, one of the bags weighs 78 pounds; what does the other weigh?

9. A cow and a horse are worth $137; the cow is worth $67; what is the horse worth?

10. A man owed $172; he paid $86; how much does he still owe?

LESSON XII.

BLACKBOARD EXERCISES.

To find the difference between 100 *and any number less than* 100 (*whose unit place is occupied by a significant figure.*)

MODEL OPERATION.

100	100	100	100	100	100	100	100	100
28	29	80	61	37	98	86	31	46

8. Rule.—IV. *Diminish the difference of the tens by one, and annex the unit figure; thus:* 8, 2; 72: 8, 1; 71, *&c.*

PROBLEMS.

1. There were 100 peach-trees in one orchard; 81 of them were blown down during a storm; how many remained standing?

2. A butcher bought 100 sheep, and killed 37; how many were left alive?

3. A grocer had a tub of butter containing 100 pounds; he sold 48 pounds to Mr. Wilson; how much remained?

4. Charles had 100 cents, and gave 97 cents for knife; how much had he left?

5. A man had $100, and paid $84 for a yoke of oxen; how much had he left?

6. A drover bought 100 sheep, and then sold 36 of them; how many had he left?

SUBTRACTION.

(1.)	(2.)	(3.)	(4.)
300—46	900—66	600—73	500—67
400—37	500—28	400—27	300—49
600—67	600—72	400—67	400—89
500—13	800—93	500—98	500—23
900—67	600—23	300—37	600—83
400—39	400—72	400—48	900—79

(5.)	(6.)	(7.)	(8.)
400—37	400—37	900—37	500—78
600—19	600—87	400—67	600—37
600—87	400—69	800—23	400—17
600—24	400—38	900—67	300—86
300—78	600—89	500—89	100—15
600—88	400—16	300—72	300—27

LESSON XIII.

BLACKBOARD EXERCISES.

MODEL OPERATION.

341	443	544	645	346	467	948	649
88	74	94	54	97	83	73	83

212+41=253.

Form.—88 from 300 leave 212; 212 and 41 are 253.

9. Rule.—V. *Take the subtrahend from the hundreds of the minuend, and to the remainder add tens and units of the minuend;* thus, 212+41=253; 813 *+42=855, &c.*

PROBLEMS.

1. Sold a quantity of wool for $379, and received in payment flour worth $97; how many dollars remain due?

2. James has 128 marbles, and John has 67; how many more has James than John?

3. A man purchased a watch for $169, and paid $84; how much did he still owe?

4. Michael had 284 cents, and lost 46; how many had he left?

5. A man sold 123 sheep from a flock consisting of 194; how many sheep remained.

6. Mr. Blore bought a horse for $135, and sold it for $346; how much did he gain?

7. A merchant bought a quantity of goods for 686 dollars, but as they were damaged he was obliged to sell them for 393 dollars; how much did he lose?

8. From a vessel containing 217 gallons, 87 gallons leaked out; how much remained in it?

9. A merchant bought a quantity of silk for $87, and sold it for $155; how much did he gain by the bargain?

10. Paid 212 dollars for a quantity of sugar, and 41 dollars for some molasses; how much more did the sugar cost than the molasses?

EXERCISES.

11. How many are $10+5+6-8-7+6-3+8+$

9−6+8+3−9+8−3−7+6+8−9+6+7−8+6 +9+8−3+8−6+6+8+6−8+9+6+3?

12. How many are 13+5+3−6+8−3+6+8+3−7−6+8+9+6−3+8+7−3+6+8−3+8+6 8+6+9+3−2+8−3+6−8+3+7−8−6+8+6 +3?

13. How many are 4+7+8+9−3+6−8+8−3 +4+8−3+7−6+7+8−3−8−7+6+7+8−6−8−7+3+6+8+3−7−6−8+7+6−2+7−2+7?

LESSON XIV.

BLACKBOARD EXERCISES.

MODEL OPERATION.

515	323	444	515	616	717	818	919
66	73	74	96	83	75	95	39
434+15=449							

Form.—515−66=what? 500−66=434; 434+15=449.

PROBLEMS.

Containing not more than two elementary questions.

1. A man sold a cow for $20 and a calf for $12; he received in payment $19; how much was still due?

ANALYSIS.*

Analytical Steps.—1. *Find the amount for which he sold the cow and the calf.*

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* Note for the Teacher.—Have the formulas recited until the pupils *can give them readily* and accurately. N. B. The analytical steps should *always be distinctly* stated in every problem.

Arith. Form.—If he sold the cow for $20, and the calf for $12, he sold both for the sum of these quantities, which is $32.

Anal. Step.—2. *Find the amount due.*

Arith. form.—If a man sold a cow and a calf for $32, and he received in payment $19 there must be due the difference between these amounts, which is $13.

Conclusion.—Therefore $13 were still due.

2. A farmer bought a horse for $87, and a wagon for $50; he exchanged both for 2 yoke of oxen worth $200; how much did he gain?

3. A man bought a yoke of oxen for $97; their keeping cost $13; he sold them for $125: did he gain or lose, and how much?

4. A box of raisins was bought for $7, a firkin of butter for $28; both were sold for $40: how much was gained?

5. A jeweler bought a watch for $86, a chain for $36; he sold both for $175: how much did he gain by the bargain?

6. Henry and George started from the same place, and traveled in the same direction; Henry traveled 160 miles, and George 78 miles; how far apart were they?

7. Two boats sail from the same place and in the same direction; one sails at the rate of 190 miles a day; the other, at the rate of 97 miles: how far will they be apart at the end of the day?

8. A father is 90 years of age, and is 37 years older than his oldest son; how old is his son?

EXERCISES.

(1.)

467—39
672—48
391—67
298—41
674—31
467—28

(2.)

483—67
483—68
382—42
673—83
413—27
414—67

(3.)

834—46
437—24
638—87
421—67
434—48
648—89

(4.)

671—82
915—67
498—27
864—68
372—89
672—48

(5.)

416—37
413—67
830—69
413—88
724—67
483—88

(6.)

673—47
414—57
387—89
597—87
683—64
836—78

(7.)

416—87
573—64
387—89
413—76
487—38
416—43

(8.)

918—78
347—88
417—67
394—83
764—73
418—95

LESSON XV.

1. James had 40 cents and paid 12 for a whistle and 25 for a knife; how many had he left?

Anal. Step.—1. *Find how much he paid for the whistle and the knife.*

Arith. Form.—If James paid 12 cents for a whistle and 25 cents for a knife, he paid for both the sum of these numbers, which is 37 cents.

Anal. Step.—2. Find how many cents he had left.

Arith. Form.—If James had 40 cents, and paid 37 cents for a whistle and a knife, he had left the difference of these numbers, which is 3 cents.

Conclusion.—Therefore he had three cents left.

2. Henry had 97 nuts in a basket; he put 12 in one pocket, and 13 in the other; how many nuts were left in the basket?

3. A school-boy paid 56 cents for an atlas, and 30 cents for an arithmetic; he gave the shopkeepers a dollar bill: how much change should he receive?

4. A drover bought 190 sheep of three farmers; he bought 27 of the first, and 28 of the second; how many did he buy of the third?

5. Lucy had 9 oranges, her mother gave her 11 more, and her father enough to make 40; how many did her father give her?

6. A man had 88 dollars, and gave 28 dollars for a cow, and 14 dollars for a calf; how much money had he left?

7. How many are 137—84? 97—68? 149—68? 137—57? 138—67? 168—78? 148—39? 163—47? 136—83? 126—72? 137—27?

8. How many are 136—72? 146—38? 97—27? 163—124? 150—35? 157—27? 138—69? 137—67? 168—134? 168—27? 186—39?

9. How many are 139—67? 186—79? 186—38? 171—69? 184—97? 68—27? 146—97? 167—39? 158—46? 173—67?

10. How many are 144—39? 183—167? 127—

117? 137–86? 137–56? 147–16? 132–73? 127–67? 133–87? 169–87?

11. How many are 137–86? 143–67? 196–67? 143–68? 176–94? 138–73? 96–28? 173–69? 183–67?

12. How many are 171–89? 146–89? 137–68? 143–87? 143–69? 137–78? 164–89? 137–87? 186–97?

13. Henry bought a slate for 25 cents, some pencils for 11 cents; he had 80 cents at first; how much has he left to spend?

14. John had 50 apples; he gave 17 to his mother, and 15 to his sister; how many had he left?

15. A man bought one barrel of pork for $18, and another for $17; he sold both for $46; how much did he gain?

16. George spent 19 cents for candy, and 17 cents for fruit; how much more will he have to spend to make 50 cents?

17. Two men bought a horse, the one paying 110 dollars, and the other 25 dollars less; how much did both pay?

18. A man sold a firkin of butter for $17, and a cheese for $19; and received in payment a barrel of flour worth $13; how much remains due?

19. Solomon gave 13 cents for some oranges, 27 cents for some cake, and had 47 cents remaining; how much had he at first?

20. Stoddard at a game of marbles won 13, and lost 25, and then had 17 remaining; how many had he at first?

21. A farmer bought a horse for $67, and exchanged it for a yoke of oxen, which he sold for $87; how much did he gain by the operation?

22. A box of raisins was bought for $5, a firkin of butter for $18; and both were sold for 20 dollars; how much was lost?

23. A drover bought some sheep for $40; some cattle for $130; he sold them all for $200; what was his gain?

24. A jeweler bought a watch for $83 and a chain for $15; he sold them both for $105; what was his gain?

25. A merchant bought a hogshead of molasses for $53; he paid $5 cartage; for how much must he sell it to gain $23?

MULTIPLICATION.

LESSON XVI.

BLACKBOARD EXERCISES.

Model Operation.

90	20	50	40	70	60	90	80
3	7	6	5	8	9	1	4
—	—	—	—	—	—	—	—
27; 270.							

Form.—3 times 9 tens are 27 tens; 27 tens are equal to 270 units.

10. Rule.—I. *Write the product of the tens and annex a cipher;* thus 27, 270; 6, 60, &c.

Study.—The pupil should write ten or more exercises on his slate like the model, and begin at the left hand, and repeat the product of each combination (See Rule); thus, 27, 270; 6, 60, &c.

The lesson should be repeated not less than ten times, commencing at the right and left alternately.

Recitation.—The teacher should write on the blackboard exercises like the above, to be recited in the same manner as the pupil is required to study them. The class should also be thoroughly questioned, the answers being given instantly, sometimes with, and sometimes without, using the blackboard.

PROBLEMS.

Containing but one Elementary Question.

1. What will 80 pounds of rice cost at 5 cents a pound?

Arith. Form.*—*Since one pound of rice costs* 5 *cents,* 80 *pounds will cost* 80 *times* 5 *cents, which is* 400 *cents.*

2. What will 50 spelling books cost at 7 cents apiece?

3. What will 40 pounds of sugar cost at 12 cents a pound?

4. What will 90 oranges cost at three cents apiece?

5. What will 40 pine-apples cost at 11 cents apiece?

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* Note.—We give the following formula for the use of those who may prefer it.

Arith Form.—80 *pounds cost* 80 *times as much as* 1 *pound;* 1 *pound costs* 5 *cents, therefore* 80 *pounds cost* 80 *times* 5 *cents, which are* 400 *cents.*

6. What will 30 pears cost at 4 cents apiece?

7. What will 70 quarts of milk cost at 5 cents a quart?

8. What will 500 lemons cost at 7 cents apiece?

9. What will 800 pounds of veal cost at 4 cents a pound?

10. At 11 cents a pound what will 300 pounds of cinnamon cost? 200 pounds? 80 pounds? 600 pounds?

EXERCISES.

NOTE.—In the following and similar exercises, the sign affects only the number immediately following it.

11. How many are $6+3-4\times8-25+5\times4-40+8+17-6-50+4+8-20\times7-3+3+12-10\times3-27+4\times8-15$?

12. How many are $12+8-15\times4-10+7-10\times9-60\times4\times4+2-50+9\times8+8-40-27+17\times1-20-4\times3+12-18\times6-36$?

13. How many are $4+19-7-10\times4-20-4\times8+8+20+50-25-23\times9+7-3-6-6-30\times1-7+5+8-15-9+7-3$?

14. How many are $4+8-6\times4+5-9+30-25+37+42-100\times12+12-17+30+3-15-1\times4+17-8+7-6+8+9+7+34-27$?

15. How many are $5\times8+3-41\times8+4-10-8\times11+30-3+53-13-87+9+23-15-8\times9+7+34+5+9-5-15-8-11$?

RECITATION.—(See Lesson IX, E. 2.)

LESSON XVII.

BLACKBOARD EXERCISES.

MODEL OPERATION.

98	72	64	72	72	71	62	27	82
4	4	8	3	6	8	3	4	4
—	—	—	—	—	—	—	—	—

36, 32; 392

FORM.—4 times 90 are 360; 4 times 8 are 32; 360 and 32 are 392.

11. **Rule.**—II. *Write the product of each figure separately, and add the tens of each product to the units of the preceding product; thus*, 36, 32; 3 (6+3) 2; 392: 16, 6; 166.

PROBLEMS.

1. What will 8 yards of cloth cost at 21 cents a yard? At 31 cents? At 61 cents? At 91 cents?

2. If a quantity of provisions will last 86 men 81 days, how long will it last 1 man?

3. If 9 men can do a piece of work in 31 days, how long will it take 1 man?

4. If 7 cows can eat a ton of hay in 31 days, how long will it take one cow to eat it?

5. If 9 horses eat 20 bushels of oats in 20 days, how long will it take one horse to eat the same quantity?

6. If 30 horses eat 24 bushels of oats in 8 days how many horses will be required to eat as many in *one day*?

7. If 91 men eat 40 cheeses in 6 days, how many men will be required to eat the same quantity in one day?

8. If 70 cows can eat 9 tons of hay in 8 days, how many cows will be required to eat the same quantity in one day?

EXERCISES.

(1.)	(2.)	(3.)	(4.)	(5.)
36 × 6=	47 × 3	41 × 4	67 × 3	36 × 2
42 × 4=	68 × 4	76 × 2	27 × 4	68 × 7
37 × 3=	97 × 3	38 × 2	67 × 3	49 × 3
27 × 2=	86 × 7	77 × 3	31 × 9	73 × 2
64 × 5=	41 × 3	46 × 4	37 × 8	67 × 2
28 × 7=	76 × 8	24 × 3	67 × 3	93 × 4

(6.)	(7.)	(8.)	(9.)	(10)
27 × 7	56 × 3	24 × 7	56 × 3	41 × 6
24 × 6	72 × 3	67 × 8	72 × 3	37 × 8
34 × 3	78 × 8	38 × 3	91 × 6	27 × 4
72 × 8	72 × 4	91 × 6	84 × 3	76 × 4
67 × 3	89 × 6	27 × 4	67 × 4	84 × 6
47 × 6	76 × 3	67 × 4	37 × 5	47 × 8

LESSON XVIII.

BLACKBOARD EXERCISES.

MODEL OPERATIONS.

68	89	46	58	69	72	86	93
3	4	5	6	7	8	9	8
—	—	—	—	—	—	—	—
18, 24; 204							

FORM.—3 times 60 are 180; 3 times 8 are 24; 180 and 24 are 204.

N. B.—The class should *write* the numbers (see Rule 11.) on the slate or blackboard, until they are given readily, after which they should be recited.

PROBLEMS.

1. What will 2 cows cost at 41 dollars each?
2. What will 3 sheep cost at 11 dollars each?
3. What will 51 pencils cost at 7 cents apiece?
4. What will 42 nutmegs cost at 3 cents apiece?
5. What will 53 apples cost at 4 cents apiece?
6. What will 25 arithmetics cost at 12 cents apiece?
7. What will 37 pens cost at 3 cents apiece?
8. What will 35 oranges cost at 8 cents apiece?
9. What will 56 lemons cost at 7 cents apiece?
10. How many are 8 times 73? 63? 52? 72? 81? 73? 62?
11. How many are 9 times 83? 52? 63? 93? 42? 72? 62?
12. How many are 3 times 92? 63? 72? 43? 52? 81? 73?
13. How many are 11 times 72? 42? 13? 83? 73? 63? 43?
14. How many are 12 times 83? 63? 93? 42? 82? 52? 81?
15. How many are 9 times 72? 46? 37? 54? 87? 63? 24? 47?
16. How many are 7 times 42? 83? 47? 29? 63? 82? 74? 49?

LESSON XIX.

BLACKBOARD EXERCISES.

MODEL OPERATION.

374	834	327	446	974	986	981
4	6	8	4	4	8	8
12, 28, 16; 1496						

Form.—4 times 300 are 1200, 4 times 70 are 280; 4 times 4 are 16: 1200+280+16=1496.

PROBLEMS.

1. At 12 cents apiece how much will 12 pineapples cost? 24 pineapples? 34? 23? 44? 64? 24? 84? 64?

2. At 8 cents apiece how much will 25 melons cost? 95 melons? 55? 85? 15? 75? 105? 65? 94? 84?

3. What will 96 lemons cost at 4 cents apiece? 75 lemons? 36 lemons? 23 lemons? 76 lemons? 85 lemons?

4. At 7 dollars a hundred, how much will 14 hundred feet of cedar boards cost? 34? 26? 46? 24? 75? 55? 95?

5. At 8 dollars a barrel what will 25 barrels of pork cost? 35? 24? 53? 86? 76? 25? 94? 64? 84? 16? 66? 56? 105?

6. How many are 4 times 24? 14? 94? 84? 64? 74? 54?

7. How many are 3 times 75? 65? 25? 95? 45? 85? 35?

8. How many are 4 times 86? 76? 36? 26? 66? 96? 85?

9. How many are 5 times 75? 76? 83? 73? 65? 25? 86?

10. How many are 6 times 85? 63? 93? 75? 24? 64? 67?

11. How many are 7 times 33? 73? 63? 83? 95? 55? 76?

12. How many are 8 times 44? 64? 84? 33? 63? 24? 86?

13. How many are 9 times 104? 76? 43? 84? 72? 63? 44?

14. How many are 11 times 103? 63? 72? 41? 64? 24? 83?

15. How many are 12 times 34? 64? 26? 16? 13? 14? 86?

LESSON XX.

BLACKBOARD EXERCISES.

169	389	479	569	659	749	839	929
4	9	6	3	9	11	4	12

4, 24, 36; 676

PROBLEMS.

1. 19 turkeys will cost how much at 8 dimes apiece? At 7 dimes? At 6 dimes? At 9 dimes?

2. If one paper of candies costs 37 cents, how much will 7 papers cost? 5 papers? 12 papers? 8 papers? 3 papers?

3. If 1 bushel of wheat costs 75 cents, how much will 3 bushels cost? 8 bushels? 5 bushels? 7 bushels?

4. How much will 72 barrels of pork cost at $7 a barrel? At $9? At $8? At $5? At $12?

5. How much will 9 tons of hay cost at $17 per ton? At $15? At $19? At $27? At $16? At $18? At $23?

6. In one year there are 12 months; how many months are there in 13 years? In 27 years? In 43 years? In 87 years?

7. A mechanic earns $7 in a week; how many dollars can he earn in 17 weeks? In 27 weeks? In 83 weeks? In 45 weeks?

8. How many are twice 147? 167? 137? 196? 143? 127?

9. How many are 3 times 227? 265? 283? 247? 234? 237?

10. How many are 4 times 346? 337? 384? 349? 367? 346?

11. How many are 5 times 437? 467? 443? 489? 453? 444?

12. How many are 6 times 527? 538? 543? 589? 657? 547?

13. How many are 7 times 634? 623? 646? 689? 683? 663?

14. How many are 8 times 743? 764? 797? 783? 767? 743?

15. How many are 9 times 872? 867? 893? 848? 833? 887?

16. How many are 11 times 938? 963? 921? 946? 954? 933?

17. How many are 12 times 829? 746? 633? 580? 420? 388?

LESSON XXI.

PROBLEMS.

Containing not more than two Elementary Questions.

1. A man bought 27 cords of wood at $8 per cord, and gave in exchange produce worth $89, how much remained unpaid?

ANALYSIS.

ANAL. STEP.—1. *Find the cost of the wood.*

ARITH. FORM.—Since 1 cord costs $8, 27 cords will cost 27 times $8, which is $216.

2. *Find balance unpaid.*

ARITH. FORM.—If the wood costs $216 and the produce was worth $89, the balance unpaid will be the difference of these quantities, which is $127.

Therefore, the balance unpaid is $127.

2. A man bought 27 sheep at $4 apiece, and gave in payment a note for $47, and the remainder in cash; how much cash did he pay?

3. A tailor had a piece of broadcloth containing 42 yards, he cut from it 17 yards; how much was the remainder worth at $4 a yard?

4. A man sold 17 sheep at $5 apiece, he received $37 in cash, and the rest in wood; how much did he receive for his wood?

5. A man started to make a journey of 279 miles; he traveled 4 days at 38 miles a day; how far was he from the end of his journey?

6. A farmer had 108 bushels of potatoes; he sold 47 bushels; how much was the remainder worth at 7 shillings a bushel?

7. A tailor has a piece of cloth containing 57 yards; he cuts from it 13 suits containing 4 yards each; how many yards remain?

8. I have 337 yards of calico, how much remains if 37 dresses, each containing 9 yards, are cut off?

9. Charles bought 93 oranges at 4 cents apiece; he paid toward them 178 cents; how much did he still owe?

10. A man earned 537 cents in a week; he paid 39 cents a day for board and washing; how much had he left at the expiration of the 7 days?

11. A man owing $189 gave in part payment 23 sheep at $4 per head; how much did he still owe?

12. I *bought* 28 sheep at $6 per head, and a span

of horses for $297; how much more did the horses cost than the sheep?

13. A man paid $189 toward 147 bushels of wheat at $2 a bushel; how much did he still owe?

14. I have 7 bags of grain; 6 of them contain 89 pounds each; they all contain 722 pounds; how many pounds does the seventh contain?

15. I have a cistern containing 189 gallons; the faucet will discharge 2 gallons every minute; if the faucet is left open 83 minutes, how much water will remain in the cistern?

16. At 2 cents a foot how much will 12 boards cost, each board containing 13 feet?

17. What will 17 dozen of eggs cost at 2 cents apiece?

18. How many shillings will 3 pieces of cloth cost at 3 shillings a yard, if each piece contains 37 yards?

19. What will 7 dozen arithmetics cost at 2 shillings apiece?

20. How many pounds in 8 bags of wheat, each bag containing 2 bushels of 60 pounds?

21. If I buy 6 barrels of pork, at $16 a barrel, and sell it for $105, how much do I gain?

22. If I buy 12 pounds of sugar at 16 cents per pound, and sell it for 205 cents, how much do I gain?

23. In an orchard are 26 peach trees, and 6 times as many apple trees; how many trees in the orchard?

24. In an orchard there are 8 rows of trees, and 125 trees in each row; how many trees in the orchard?

25. If 8 horses eat 12 bushels of oats in 6 days, how many bushels will they eat in 32 days?

LESSON XXII.

BLACKBOARD EXERCISES.

To find the product of any two numbers between eleven and twenty.

MODEL OPERATION.

17	12	17	19	16	17	14	13	18
13	16	15	11	19	18	15	13	13
—	—	—	—	—	—	—	—	—
1, 10, 21; 221								

Form.—Ten times 17 are 170; 3 times 17 are 51; 170 and 51 are 221.

12. **Rule.**—*Write the product of the tens, the sum of the units, and the product of the units separately; add the tens of each number to the units of the preceding number;* thus, (Ex. 1.), 1, 10, 21; 221: 1, 11, 30; 240: 1, 8, 12; 192: &c.

Note for the Teacher.—The pupil at first should *write* the numbers (see Rule) on the slate or the blackboard, until they can give the product readily; then the work should be done mentally.

EXERCISES.

1. How many are 13 times 12? 15? 16? 17? 13? 19? 183?

2. How many are 14 times 11? 12? 13? 14? 15? 16? 17?

3. How many are 15 times 13? 16? 14? 18? 19? 11? 12?

4. How many are 16 times 14? 13? 15? 17? 16? 18? 12?

5. How many are 17 times 13? 11? 15? 16? 18? 12? 17?

6. How many are 18 times 12? 15? 16? 11? 17? 18? 19?

7. How many are 11 times 15? 19? 16? 18? 13? 11? 17?

8. How many are 19 times 16? 11? 18? 14? 13? 15? 12?

9. How many are 12 times 15? 16? 19? 13? 14? 17? 16?

BLACKBOARD EXERCISE.

To multiply any number between 20 *and* 100 *by any number between* 10 *and* 20.

MODEL OPERATION.

87	71	63	47	96	34	47	52
17	15	13	19	18	11	14	17
—	—	—	—	—	—	—	—
8, 63, 49; 1479							

FORM.—Ten times 87 are 870; 7 times 80 are 560; 7 times 7 are 49; 870+560+49=1479.

13. Rule.—IV *Write the product of the tens, the product of the units of the multiplier and the tens of the multiplicand, increased by the unit figure of the multi-*

plicand, and the product of the units, separately; add the tens of each number to the units of the preceding number; thus: (Ex. 1.), 8, 63, 49; 1479: 6, 21, 9; 819: 4, 26, 12; 672: &c.

EXERCISES.

1. How many are 11 times 72? 84? 36? 27? 46? 36?

2. How many are 12 times 64? 38? 56? 29? 44? 82?

3. How many are 13 times 29? 36? 48? 53? 64? 73?

4. How many are 14 times 33? 86? 98? 77? 68? 28?

5. How many are 15 times 46? 38? 96? 74? 28? 33?

6. How many are 16 times 27? 54? 32? 91? 77? 66?

7. How many are 17 times 86? 42? 68? 34? 22? 53?

8. How many are 18 times 43? 36? 26? 68? 95? 58?

9. How many are 19 times 36? 58? 88? 74? 27? 68?

(10.)	(11.)	(12.)	(13.)
37 × 13	49 × 16	27 × 15	21 × 17
27 × 12	87 × 13	67 × 13	72 × 16
63 × 14	69 × 16	78 × 16	49 × 13
57 × 13	56 × 13	47 × 13	78 × 16
69 × 11	97 × 16	29 × 19	97 × 18

DIVISION.

LESSON XXIII.

BLACKBOARD EXERCISES.

1. 2) 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

2. 3) 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

3. 4) 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

N. B. Complete the exercises by using each of the nine digits as a divisor.

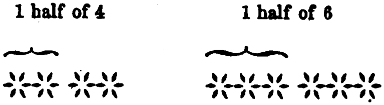
NOTES FOR THE TEACHER.—1. The above and following blackboard exercises are to be studied and recited in the same way as the similar exercises in addition, subtraction, and multiplication.

2. The remainders should be given, thus (Ex. 2.): No times and 1 over; no times and 2 over; once; one and one over; one and 2 over; &c. It is very important that these exercises should be LEARNED SO THOROUGHLY that the quotients and remainders can be recited with the utmost promptness.

PROBLEMS.

1. What is meant by 1 *half?* ANS.—*One of the two equal parts into which a unit or a number of units is divided.*

ILLUSTRATION.



1 half of 1.

2. How many is 1 half of 6 stars? Of 18 stars? Of 20 stars?

3. How many is 1 half of 12 oranges? Of 16 apples? Of 24 pears?

4. How many is one half of 32 pounds? Of 14 gallons? Of 26 inches?

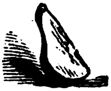
5. What is meant by 1 third? Ans.—*One of the three equal parts into which a unit or number of units are divided.*

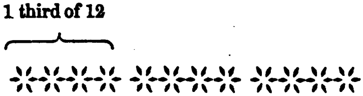
6. What is meant by 1 fourth? By 1 sixth? By 1 ninth?

7. What is meant by 1 eleventh? By 1 fifteenth?

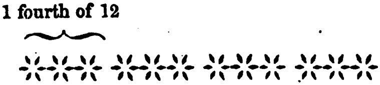
8. How many in 1 third of 12 stars? In 1 fourth of 12 stars?

ILLUSTRATION.





1 third of 1



9. How many in 1 third of 6 stars? Of 9 *stars? Of 12* stars?

10. What is meant by 1 fifth? By 1 seventh? By 1 ninth?

11. How many in 1 seventh of 14? Of 21? Of 28?

12. How many in 1 eighth of 16? Of 24? Of 96?

13. How many in 1 eighth of 24? Of 36? Of 88?

14. How many in 1 ninth of 27? Of 72? Of 108?

15. How many in 1 eleventh of 99? Of 88? Of 121?

PROBLEM.—If 15 apples are equally divided among 3 boys, how many apples will each boy receive?

ARITH. FORM.*—*If three boys have* 15 *apples, one boy will have one third of* 15 *apples, which is* 5 *apples.*

NOTE.—Let the pupil be taught that one *third* of a thing or number is the same as the quotient when the thing or number is divided by 3; hence, for brevity, we say, "one third of a thing or of a number," instead of a "thing or a number divided by three."

16. If 9 peaches cost 27 cents, what is the cost of 1 peach?

17. If 8 lead-pencils cost 24 cents, how much is that apiece?

18. I pay 32 dollars for 8 cords of wood; how much is that per cord?

---

[*] The following formula is preferred by some :—

FORM.—One boy will have one third as many apples as three boys; 3 boys have 15 apples, therefore one boy will have one third of 15 apples, which is 5 apples.

19. A man earned $36 in 9 days; how much is that per day?

20. 8 gold chains cost $64; what is the cost of each?

21. 5 barrels of apples are worth $35; what is the value of 1 barrel?

22. In 7 hours a man rode 84 miles; how far was that an hour?

23. 9 pounds of sugar cost 81 cents; how much is that a pound?

24. In 9 rows of trees there are 72 trees; how many trees in each row?

25. A man traveled 72 miles in 6 hours; how far did he travel in one hour?

26. 8 bins of equal size contain 72 bushels of wheat; how many bushels does each contain?

27. If 81 cents be paid for 9 dozen eggs, what will be the cost of 1 dozen?

28. 11 tons of hay are worth $77; what is the value of one ton?

29. 12 men can perform a certain piece of work in 48 days; what part of it can 1 man perform?

30. If 7 men can eat a cheese in 7 days, what part of it can 1 man eat in the same time?

31. One man can perform a piece of work in 48 days; how long will it take 12 men to do it?

32. One man can do a piece of work in 56 days; how long will it take 8 men?

33. One man can build a barn in 63 days; how long will it take 9 men to build it?

34. One man can build a house in 108 days; how long will it take 9 men to build it?

35. One man can do a job of work in 45 days; in what time can 5 men do it?

EXERCISES.

36. How many are 25 divided by 3? Ans.—8 *and* 1 *over*. By 4? Ans.—6 *and* 1 *over*. By 2? By 7? By 9? By 11? By 8? By 6?

37. How many are 50 divided by 7? By 3? By 2? By 5? By 6? By 4? By 11? By 8? By 9? By 12? By 1? By 20? By 10?

38. How many are 45 divided by 2? By 4? By 3? By 5? By 8? By 7? By 9? By 11? By 10? By 18? By 12? By 25? By 40?

39. How many are 63÷8? By 7? By 6? By 5? By 4? By 3? By 2? By 1? By 20? By 30? By 9? By 10? By 11? By 12?

40. How many are 70 divided by 11? By 9? By 7? By 5? By 3? By 1? By 2? By 9? By 4? By 8? By 12? By 36? By 22?

(41.)

47÷6
37÷8
50÷10
72÷12
96÷11
47÷6

(42.)

47÷8
67÷4
39÷7
67÷8
69÷8
59÷6

(43.)

57÷4
97÷9
68÷4
37÷5
67÷8
37÷4

(44.)

63÷9
47÷8
63÷8
97÷4
63÷7
69÷8

LESSON XXIV.

BLACKBOARD EXERCISES.

MODEL OPERATION.

1. 2)	100	120	140	160	180	200	220	240	260
2. 3)	120	150	180	210	240	270	300	330	360
3. 4)	120	160	200	240	280	320	360	400	480

Note.—Complete the table through the digits.

Study and Recitation.—(See Lessons I. and VIII.)

PROBLEMS.

1. If 50 quarts of molasses cost 300 cents, what will one quart cost?

2. If 120 lemons cost 360 cents, how much will one cost?

3. If 80 pineapples cost 960 cents, how much will one pineapple cost?

4. If 70 barrels of pork cost $560, how much will one barrel cost?

5. If one barrel of flour will support 20 persons one week, how many persons will it support for four weeks?

6. How much will one cord of wood cost, if 20 cords cost $400?

7. If 25 sheep cost $75, what will one sheep cost?

8. A person divided 72 peaches among 9 boys; how many did each receive?

9. 98 marbles were equally divided among 2 boys; how many did each boy receive?

10. A boy had 96 chestnuts, which he divided among 3 playmates; how many did each playmate receive?

EXERCISES.*

11. How many are $20+13\div11\times4+18\div10\times9-7\div2+8+26-14\div2+4+8-7\div2+10\div4\times8\div10-4$?

12. How many are $3\times9+3\div2+3+4\div11+26-2-16\div5\times76\div1-92\div10-5\times46-36\div10+8-7\times4$?

13. How many are $40\div4+17+4-8-3-18\times9+36-50\times9-18+7\times4-100+89-72-15\times8\div2-8$?

14. How many are $2+37+8-42+6\div11\times3+3-4\times9\div9+6+8\div2+27\div7\times36\div9+16\div12+10\times6\div24$?

15. How many are $3\times16\div12\times27\div9\times40-80\div4\div10\div5\times11+80\div2-19\div8-3\times9+17+6-11$?

16. How many are $11\times2+8\times2+40\div10-10+78-6\div12\times13\div2-9\div5+37-2$?

17. How many are $4\times9+7+2\div9\times25-100\times4+100+37-6+8+7\div2-3\div8$?

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* *N. B.*—*The teacher* should frequently dictate exercises like the following.

LESSON XXV.

BLACKBOARD EXERCISES.

MODEL OPERATION.

4) 168   208   244   284   328   364   408   444

Form.—(Ex. 1.) 4 is contained in 160, 40 times; 4 is contained in 8, 2 times; 40 and 2 are 42.

Study and Recitation.—(See Lessons I. and VIII.)

PROBLEMS.

1. At 5 cents each, how many lemons can I buy for 20 cents?

Form.[*]—*Since 5 cents will buy one lemon, as many lemons can be bought for* 20 *cents as* 5 *cents are contained times in* 20 *cents, which is* 4 *times.*

2. How many boxes of cheese, at 5 dollars a box, can be had for $30?

3. If I receive $12 interest in one year, in how many years shall I receive $132 interest?

4. How many pencils can I buy for 360 cents, if one pencil costs 3 cents?

5. At 7 cents a pound, how many pounds of lead may be had for 840 cents?

6. How many barrels of apples, at $3 a barrel, can be bought for $183?

Note.[*]—The following formula may be used instead of the above :—

Form.—*Since each* 5 *cents will buy* 1 *lemon, as many lemons can be bought for* 20 *cents, as the number of times that* 5 *cents can be taken from* 20 *cents, which is* 4 *times, &c.*

7. A man sold a wagon for $128, and took his pay in cider at $3 a barrel; how many barrels did he receive? ANS.—42⅔ *bar.*

8. How many barrels of flour, at $5 a barrel, can be bought for $155?

9. How many pounds of steak, at 8 cents a pound, can be bought for 320 cents?

10. How many pounds of sugar at 9 cents a pound can be bought for 729 cents?

11. How many sheep, at $9 a head, can be bought for $639? For $369? For $540?

12. If 25 sheep cost $75, what will one sheep cost?

13. At 15 cents a pound, how many pounds of coffee can be bought for 60 cents? For 90 cents? For 30 cents? For 135 cents?

14. One man can do a piece of work in 726 days; in what time can 6 men do it?

15. One man can eat a quantity of provisions in 279 days; in how many days can 3 men eat the same?

16. How many men, at $8 a week, can be hired for $728? For $488? For $248?

17. If I travel 120 miles in 6 days, how far do I travel in one day?

18. I have a farm containing 126 acres; how many lots of 6 acres each does it contain?

19. At 13 cents a pound, how many pounds of cheese can be bought for 39 cents?

20. An orchard contains 368 trees; in each row *there are 8 trees;* how many rows in the orchard?

LESSON XXVI.

PROBLEMS.

1. If 20 men accomplish a certain piece of work in 1 day, how many men will be required to do it in 4 days?

2. If 48 men use 2 barrels of flour in one week, how many men will use the same in 6 weeks?

3. If 75 persons eat 80 loaves of bread in one week, how many persons will it require to use the same in 5 weeks?

4. If 3 yards of cloth are enough for a coat, how many coats can be made from 33 yards?

5. A man traveled 7 miles in one hour; at that rate, how long would it take him to travel 63 miles?

6. For 49 cents how many pounds of sugar can be bought at 7 cents a pound?

7. For 240 cents how many oranges can be bought at 3 cents apiece? At 4 cents? At 6 cents?

8. For 360 cents how many pounds of sugar can be bought at 9 cents a pound? At 6 cents? At 12 cents?

9. If 12 men can build 240 rods of wall in 20 days, how many rods can one man build in the same time?

10. 12 men can build 240 rods of wall in 20 days; how many rods can they build in one day?

11. If 9 men can do a piece of work in 3 days, how long will it take 1 man to do it?

Form.—*It will take one man 9 times as long as 9 men to do the work; 9 men can do it in 3 days, therefore 1 man can do it in 9 times 3 days, or 27 days.*

12. If 8 men can do a piece of work in 3 days, how long will it take 1 man to do it?

13. If 3 men can do a piece of work in 5 days, how long will it take 1 man to do it?

14. How long will it take 1 man to do what 9 men can do in 8 days?

15. If 6 men can do a piece of work in 9 days, how many men would be required in order to do it in 1 day?

Form.—*To do it in 1 day requires 9 times as many men as to do it in 9 days; to do it in 9 days requires 6 men, therefore to do it in 1 day requires 9 times 6 men, or 54 men.*

16. If 6 men can build a wall in 12 days, how many men will it take to build it in 1 day?

17. If 6 men can build a wall in 12 days, how many days will it take 1 man to build it?

18. 8 men can earn $30 in 6 days; how many men can earn the same in 1 day?

19. 8 men can earn $30 in 6 days; how many days will it take 1 man to earn it?

20. In 9 weeks 8 persons use a barrel of flour; how long would it last 1 person?

21. In 11 weeks 22 persons use 2 barrels of flour; how long would it last one person?

22. In 11 weeks 22 persons use 2 barrels of flour; how many men would be required to consume it in one week?

LESSON XXVII.

MENTAL EXERCISES.

1. 124 are how many times 2? 4? 62? 31?

2. 134 are how many times 2? 3? Ans.—44 *and* 2 *over.*

3. 142 are how many times 2? 3? 4? 5? 6?

4. 152 are how many times 2? 3? 4? 5? 6? 7? 8? 9?

5. 162 are how many times 2? 3? 4? 5? 6? 7? 8? 9?

6. 172 are how many times 2? 3? 4? 5? 6? 7? 8? 9?

7. 182 are how many times 2? 3? 4? 5? 6? 7? 8? 9?

8. 192 are how many times 2? 3? 4? 5? 6? 7? 8? 9?

9. 200 are how many times 2? 3? 4? 5? 6? 7? 8? 9?

10. 210 are how many times 2? 3? 4? 5? 6? 7? 8? 9?

MISCELLANEOUS PROBLEMS.

Containing not more than three Elementary Questions.

11. A farmer bought a horse for $65, a saddle for 16 dollars, and a bridle for $8; what did he pay for all?

12. A lady paid $200 for a piano, $37 for a sofa, and $20 for an easy chair; what did she pay for all?

13. Julia bought 2 pieces of tape for 20 cents, 2 papers of pins for 26 cents, and a pair of scissors for 50 cents; how much did she pay for all?

14. A farmer has 115 sheep in one lot, 75 in another, and 40 in his barnyard; how many sheep has he?

15. James has 39 nuts in one pocket, and 14 less in another; how many has he in both?

ANAL. STEPS.—1. *Find how many nuts in the second pocket.*

FORM.—If James has 14 nuts less in the second pocket than in the first, he must have in the second pocket the difference between 39 nuts and 14 nuts, which is 25 nuts.

2. *Find how many nuts in both.*

If James has 39 nuts in one pocket and 25 nuts in another, he must have in both the sum of 39 nuts and 25 nuts, which is 64 nuts.

16. A man bought 40 sheep of three farmers: he bought 9 of the first, and 13 of the second; how many did he buy of the third?

17. A grocer had a tub of butter containing 48 pounds; he sold 13 pounds to Mr. Jacob Wilson, and 15 pounds to John Huntsman; how many pounds were left?

18. Mr. Squib paid $25 for a cow, and for a horse $56 more than he paid for the cow; how much did he pay for both?

19. What is the difference between the cost of 6

pieces of tape at 4 cents a piece, and of 4 yards of ribbon at 6 cents a yard?

ANAL. STEP.—1.* *Find cost of the tape.*
2. *Find cost of the ribbon.*
3. *Find the difference of cost.*

20. How many cents must a boy have, to buy 18 apples at 3 cents apiece, and 14 cakes at 2 cents apiece?

21. I bought 6 barrels of pork at eleven dollars a barrel, and sold it for $89; how much did I gain?

22. What is the cost of 13 sticks of candy at 2 cents apiece, and four pounds of raisins at 15 cents a pound?

23. If 3 dresses of 9 yards each are cut from 46 yards of calico, how many yards will be left?

24. A man started from home and traveled 5 miles an hour for 13 hours, and then set out to return, at the rate of 6 miles per hour; how far was he from home, after having traveled 19 hours?

25. Henry and John started together, and traveled in the same direction. Henry walked 3 miles an hour, and John 4 miles an hour; how far apart were they at the end of 9 hours?

26. Robert and Joseph started together from home, and traveled in the same direction; Robert traveled 2 miles an hour, and Joseph 4 miles an hour: how far were they apart at the end of 36 hours?

---

* NOTE.—Require the formula for each step until perfectly familiar, when it may be omitted.

27. The next week Robert and Joseph started together from home and traveled in opposite directions. Robert traveled 2 miles an hour, and Joseph 3 miles an hour; how far were they apart at the end of 19 hours?

28. A horse was bought for $85; he was kept 5 weeks at an expense of $4 per week, and then sold for $100; how much was lost?

29. What is 1 third of 27? Of 30? Of 36?

30. What is 1 fourth of 36? Of 48? Of 56?

31. What is 1 fifth of 20? Of 75? Of 35?

32. What is 1 eleventh of 22? Of 77? Of 99?

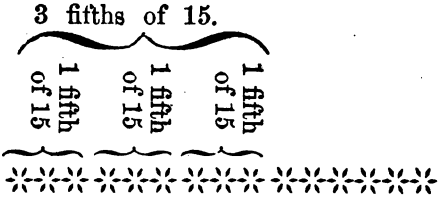
33. What is 1 thirteenth of 39? Of 52? Of 78?

34. What is 1 eighteenth of 54? Of 72? Of 126?

35. What is meant by 3 fifths? Ans.—*Three of the five equal parts into which a unit or a number of units is divided.*

ILLUSTRATION.

Three fifths of 1



36. How many stars in 3 fifths of 15 stars?

Form.—One fifth of 15 stars is 3 stars; and 3 fifths of 15 *are 3 times 3 stars*, which are 9 stars.

37. What are 2 times 1 third of 18? Of 12? Of 21?

38. What are 3 times 1 fourth of 16? Of 36? Of 48?

39. What are 3 times 1 fifth of 25? Of 45? Of 65?

40. What are 6 times 1 seventh of 21? Of 63? Of 84?

41. What are 6 sevenths of 21? Of 28? Of 84?

42. What are 5 ninths of 27? Of 72? Of 81?

43. What are 7 twelfths of 108? Of 132? Of 96?

44. If 3 yards of cloth cost $27, what will 7 yards cost?

ANAL. STEPS.*—1. *Find the cost of one yard.*
2. *Find the cost of 7 yards.*

FORM.—If 3 yards cost $27, 1 yard will cost 1 third of $27, which is $9, and 7 yards will cost 7 times 1 third of $27, which is $63.

45. If 7 oranges cost 63 cents, how much will 13 oranges cost?

46. If 9 yards of tape cost 27 cents, how much will 120 yards cost?

47. If 7 sheep cost $56, how much will 321 sheep cost?

48. Susan can solve 80 problems in 8 days; how many can she solve in 76 days?

49. I pay $84 for 12 sheep; how much must I pay for 196 sheep?

50. A man rode 75 miles in 25 hours; how far at that rate can he ride in 324 hours?

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* The teacher should first require the pupil to state the steps, and then *to give the contracted formula.*

51. A man travels 63 miles in 7 hours; how far at that rate can he ride in 72 hours?

52. If 84 dollars are paid for 21 barrels of flour, how much must be paid for 36 barrels?

53. How much will 37 hats cost, if 7 cost $21?

54. How much can a person earn in 14 days, if he can earn $48 in 24 days?

55. What must I pay for 46 apples, if 5 cost me 15 cents?

56. A farmer sold 14 calves, at the rate of 2 calves for $24; what did he receive for them?

57. 12 men can do a piece of work in 1 day, how many men will be required in order to do it in 3 days?

FORM.—*To do it in* 3 *days requires one third as many men as will be required for doing it in* 1 *day; to do it in* 1 *day requires* 12 *men; therefore to do it in* 3 *days will require one third of* 12 *men, or* 4 *men.*

58. How many men will it take to do in 9 days what 27 men can can do in 1 day?

59. How many men will it take to earn in 8 days the same money that 168 men can earn in 1 day?

60. 96 men can build a wall in 1 day; how many men will it take to build it in 3 days?

61. 75 men can dig a ditch in 1 day; how soon can 15 men dig it?

FORM.—*If* 75 *men can dig it in* 1 *day,* 15 *men will require as many days as* 15 *are contained times in* 75, *which are* 5 *times.*

62. If a certain quantity of provisions will keep a family of 18 persons 1 day, how long will it keep a family of 6 persons?

63. If 37 loaves of bread will support 27 persons one day, how many days will it support 3 persons?

64. If 72 men can build a wall in 1 day, how many days will 12 men require for building it?

65. If 1 man do a piece of work in 27 days, how soon can 3 men do the same work?

FORM.—3 *men will require* ONE-THIRD *as much time to do the work as* 1 *man;* 1 *man requires* 27 *days; therefore* 3 *men will require one*-THIRD *of* 27 *days, which is* 9 *days.*

66. If 1 man can build a barn in 135 days, how soon can 3 men do the same work?

67. How long will it take a regiment of 480 men to build a fortification, if one man can do it in 960 days?

68. A pasture will keep a horse 384 days; how long will it keep 192 horses?

69. At the rate of 3 oranges for 7 cents, how many can be bought for 63 cents?

FORM.—*Since* 7 *cents will buy* 3 *oranges, as many times* 3 *oranges can be bought for* 63 *cents as* 7 *cents are contained times in* 63 *cents, which are* 9 *times; hence* 9 *times* 3 *oranges, that is* 27 *oranges, can be bought for* 63 *cents.*

70. A boy paid 75 cents for eggs, at the rate of 7 eggs for 15 cents; how many did he buy?

71. When eggs are 9 for 25 cents, how many can I buy for 125 cents?

72. If 36 men can dig a cellar in 18 days, how many men will be required to dig it in 3 days?

FORM.—*Since* 18 *days are* 6 *times* 3 *days it will take* 6 *times as many men to do it in* 3 *days as in* 18 *days; it requires* 36 *men to do it in* 18 *days, therefore it will take* 6 *times* 36 *men, that is* 216 *men, to do it in* 3 *days; or,*

18 *days are* 6 *times* 3 *days; therefore to do the work in three days, will take* 6 *times* 36 *men, which is* 216 *men.*

73. If 4 men can dig a ditch in 40 days, how many men will be required to dig it in 5 days?

74. How long will it take 7 men to reap a field that 28 men can reap in 2 days?

75. Two ships, 168 miles apart, sail directly toward each other, one at the rate of 8 miles an hour, and the other at the rate of 4 miles an hour; how soon will they meet?

76. A man bought a number of lemons at the rate of 11 for 25 cents; how many could he buy for 250 cents?

77. When potatoes are sold at the rate of 4 barrels for $8, how many barrels must be given for 4 cwt. of sugar at $14 a hundred?

ANAL. STEPS.—1. *Find value of one barrel of potatoes.*
2. *Find entire value of the sugar.*
3. *Find number of barrels required.*

78. How many dozen of eggs, at 18 cents a dozen, will pay for 9 gallons of molasses at 36 cents a gallon?

79. When eggs are sold at 24 cents for 8, how many will it take to pay for 13 pounds of sugar, at 6 cents a pound?

80. How many gallons of molasses, at 35 cents a gallon, will pay for 70 pounds of butter at 30 cents a pound?

81. A man received 32 pounds of sugar in exchange for 20 pounds of butter at 16 cents a pound; what did the sugar cost him per pound?

82. A man bought 80 turkeys, at the rate of 4 for 3 dollars; he sold them at the rate of 20 for 11 dollars; did he make or lose, and how much?

83. A farmer bought 24 sheep for 60 dollars; at how much per head must he sell them in order to gain 12 dollars?

84. Five men bought a threshing machine for $150; they rented it 4 weeks at $10 a week, and then sold it for $100; did they make or lose, and what was each man's share?

85. 5 barrels of flour are worth $60; how many hundred weight of pork, at $18 a hundred, will 3 barrels buy?

86. 4 men agree to build a barn for $160; but when it is half built 2 of the men quit work, and the remainder complete the job; how much should each man receive?

ANAL. STEPS.—1. *Find what each man should receive for building the first half.*

2. *Find what each man should receive for building the second half.*

3. *Find the entire amount each man should receive.*

87. Four men agree to chop 25 cords of wood for $48, but when it is half done, three of the

men quit, leaving the remaining man to finish the work. How shall the money be divided?

88. If 4 men can do a piece of work in 24 days, how soon can 12 men do it?

ANAL. STEPS.—1. *Find how long it will take* 1 *man to perform it.*

FORM.*—If 4 men require 24 days, 1 man will require 4 times 24 days, which is 96 days.

2. *Find how long it will take* 12 *men.*

If it take 1 man 96 days, it will take 12 men 1 *twelfth* of 96 days, which is 8 days.

89. If 6 men can build a wall in 18 days, how many men will be required to build it in 24 days?

90. If 2 men can construct an engine in 15 months, in what time could 30 men construct it?

91. How soon can 12 men earn as much money as 9 men can earn in 32 months?

92. If 2 men can build a vessel in 30 months, how soon can 3 men build it?

93. If 9 men can plant 5 acres in 16 days, how soon can 1 man do the work?

94. How long will it take 8 men to do what 12 men can do in 6 days?

95. If 40 barrels of apples cost $320, what will 5 barrels cost?

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* Those who prefer may use the following:

FORM.—*One man requires to perform the work* 4 *times as many days as* 4 *men ;* 4 *men require* 24 *days; therefore* 1 *man requires* 4 *times* 24 *days, which are* 96 *days.*

2. *Twelve men require one twelfth as many days to perform the work as* 1 *man ;* 1 *man requires* 96 *days ; therefore* 12 *men require one twelfth of* 96 *days, which is* 8 *days.*

96. If a horse can trot 36 miles in 6 hours, how many miles can he trot in 281 hours?

97. How many pounds of veal can be bought for 96 cents, if 9 pounds cost 72 cents?

98. If 4 quarts of vinegar can be bought for 36 cents, how many quarts can be bought for 216 cents?

99. If 3 pounds of cheese cost 42 cents, what will 75 pounds cost?

100. What is the cost of 27 weeks' board, if 9 weeks' board cost $27?

101. What will 75 tons of coal cost at $80 for 10 tons?

102. If 56 cents are paid for 7 dozen of eggs, how much must be paid for 27 dozen?

103. If 12 barrels of apples cost 72 dollars, what will 7 barrels cost?

104. How much corn, at 90 cents a bushel, must be given for 27 bushels of rye at 30 cents a bushel?

105. A cistern that holds 120 gallons has a pipe by which 18 gallons can run into it in an hour, and another pipe by which 12 gallons can run out of it in the same time. When both pipes are running how long will it take the cistern to fill?

CLASSIFICATION OF FORMULAS.

Note for the Teacher.—It is very important that the pupils learn the six arithmetical formulas, five of which are used in the fundamental rules; for it is by the means of these six formulas that *all problems are solved.*

14. FORM. I.—*If I have* 12 *lemons in my pocket and* 6 *in my hand, in both I have the* SUM *of* 12 *lemons and* 6 *lemons, which is* 18 *lemons.*

15. II.—*If I have* 12 *lemons in my pocket and give away* 6, *I have left the* DIFFERENCE *between* 12 *lemons and* 6 *lemons, which is* 6.

16. III.—*If* 1 *lemon costs* 6 *cents,* 12 *lemons cost* 12 *times* 6 *cents, which is* 72 *cents.*

17. IV.—*If* 6 *boys have* 12 *lemons,* 1 *boy has one* SIXTH *of* 12 *lemons, which is* 2 *lemons.*

18. V.—*Since* 6 *cents will purchase* 1 *lemon,* 12 *cents will purchase as many lemons as* 6 *cents are contained times in* 12 *cents, which is* 2 *times.*

19. VI.—*If a boy has* 12 *lemons, and he give away* 4, *he will give away* 4 TWELFTHS *of his lemons.*

For formulas III. IV. and V. the following may be used if they are preferred:

(*a*). III. *Twelve lemons will cost* 12 *times as much as* 1 *lemon:* 1 *lemon costs* 6 *cents, therefore* 12 *lemons will cost* 12 *times* 6 *cents, which is* 72 *cents.*

(*b*). IV. *One boy must have one* SIXTH *as many apples as* 6 *boys;* 6 *boys have* 12 *apples; therefore one boy must have one* SIXTH *of* 12 *apples, which is* 2 *apples.*

(*c*). V. *Since* 6 *cents will buy* 1 *lemon, as many lemons can be bought for* 12 *cents, as the number of times that* 6 *cents can be taken from* 12 *cents, which is* 2 *times.*

N. B. It is not meant that the preceding are the best or the only formulas that can be used. The teacher ought to use his own judgment, and exercise his ingenuity in adapting them to the minds of his pupils. All the principles of the solution of problems are expressed in the *six arithmetical formulas*, no matter how those principles may be varied or combined. (See Teacher's Manual.)

REDUCTION.

LESSON XXVIII.

20. U. S. Money or currency is the currency used in the United States.

(*a*) TABLE.

10 mills (m.)	make	1 cent.	ct.
10 cents	"	1 dime.	d.
10 dimes	"	1 dollar. . . .	$.
10 dollars	"	1 eagle.	E.

NOTE—1. Dollars are separated from cents and mills by a point, (.); thus $4.732, is read 4 dollars, 73 cents and 2 mills.

2. For description of coins and for other information, see "Intermediate."

PROBLEMS.

1. How many dimes in $4?

FORM.—*Since in* 1 *dollar there are* 10 *dimes, in* $4 *there are* 4 *times* 10 *dimes, which are* 40 *dimes; therefore in* $4 *there are* 40 *dimes.*

2. How many dollars in 3 eagles? In 25 eagles?

3. How many mills in 7 cents? In 14 cents? In 35 cents?

4. How many cents in 3 dollars? In $5? In $20.

ANAL. STEPS.—1. *Reduce dollars to dimes.*
2. *Reduce dimes to cents*

5. How many mills in 4 cents? In 5 dimes? In 5 dollars?

6. How many cents in 80 mills? In 60 mills? In 120 mills?

FORM.—Since in 10 mills there is 1 cent, in 80 mills there are as many cents as 10 mills are contained times in 80 mills, which is 8 times, therefore, &c.

7. How many dollars in 60 dimes? In 400 dimes? In 480 dimes?

8. In 600 cents how many dollars? In 4800 cents?

LESSON XXIX.

21. English Money or currency is the currency used in England.

(*a*) TABLE.

4 farthings (far.)	make	1 penny. . . .	d.
12 pence	"	1 shilling. . .	s.
20 shillings	"	1 pound. . . .	£.
21 shillings	"	1 guinea. . . .	guin.

NOTE.—1. A sovereign (sov.) is a coin worth one pound.

2. For description of coins and for other information, see "Intermediate."

PROBLEMS.

1. How many farthings in 2 pounds?

ANAL. STEPS.—1. *Reduce pounds to shillings.*
2. *Reduce shillings to pence.*
3. *Reduce pence to farthings.*

2. How many pence in 3 shillings? In 7s.? In 21s.? In £4? In £3? In £12?

3. How many farthings in 3 pence? In 12 pence? In 37 pence? In 5 shillings? In 3 shillings? In 2 pounds?

4. In 3 pounds how many shillings? How many pence? How many farthings?

5. In 2880 farthings how many pounds?

ANAL. STEPS.—1. *Reduce farthings to pence.*
2. *Reduce pence to shillings.*
3. *Reduce shillings to pounds.*

6. How many shillings in 84 pence? In 108 pence?

LESSON XXX.

22. Troy Weight is used for weighing gold, silver, and jewels, and in determining the specific gravity of bodies, &c.

(*a*) TABLE.

24 grains (gr.)	make	1 pennyweight. . . .	pwt.
20 pennyweights	"	1 ounce.	oz.
12 ounces	"	1 pound.	lb.

NOTE.—3⅙ grains make a carat for weighing diamonds.

PROBLEMS.

1. How many grains in 2 pennyweights? In 3 pwt.?

2. How many pennyweights in 3 ounces? In 5 ounces?

3. How many ounces in 80 grains? In 120 pwt.?

4. What will a gold chain, weighing 12 pennyweights, cost, at 80 cents a pennyweight?

5. I have a pin weighing 4 pennyweights; what is it worth at 75 cents a pennyweight?

6. Reduce 96 ounces to pounds; to pennyweights.

LESSON XXXI.

23. Avoirdupois weight is used for all ordinary purposes.

(*a*) TABLE.

16 drams (dr.)	make	1 ounce.	oz.
16 ounces	"	1 pound.	lb.
25 pounds	"	1 quarter.	qr.
100 pounds or 4 quarters	"	1 hundredweight	cwt.
2000 pounds	"	1 ton.	T.

Note.—The gross-cwt. contains 112 pounds, or 4 quarters of 28 pounds each. The gross-ton (2240 lb.) is used at the U. S. Custom House, and the Penn. Coal Mines.

1. How many drams in 3 ounces? In 4 ounces?

2. How many ounces in 2 pounds? In 5 pounds?

3. How many pounds in 5 hundred weight? In 4 cwt.?

4. How many hundred weight in 3 tons? In 4 tons?

5. In 6 oz. 5 dr. how many dr.? In 4 oz. 3 dr.?

6. In 4 lb. 6 oz. how many ounces? In 3 lb. 2 oz.?

7. In 32 ounces how many pounds? In 96 oz.?

8. In 200 pounds how many hundred weight? In 384 pounds? In 437 pounds? In 689 pounds?

9. What will 13 pounds of flour cost at $4 per hundred weight? At $2? At $8?

10. How much will 4 cwt. of sugar come to, at 8 cents a pound? At 5 cts.? At 9 cts.?

LESSON XXXII.

24. Apothecaries' Weight is used in preparing prescriptions, but drugs and medicines are bought and sold by avoirdupois weight.

(*a*) TABLE.

20 grains (gr.)	make	1 scruple. . . .	sc. or ℈.
3 scruples	"	1 dram.	dr. or ʒ.
8 drams	"	1 ounce	oz. or ℥.
12 ounces	"	1 pound.	lb. or ℔.

PROBLEMS.

1. How many grains in 3 scruples? In 4 sc.
2. How many scruples in 11 drams? In drams?
3. How many ounces in 24 pounds? In pounds?
4. How many ounces in 96 drams? In drams?
5. In 43 drams how many scruples? In drams?

LESSON XXXIII.

25. Long Measure is used for measuring distanc

(*a*) TABLE.

12 inches (in.)	make	1 foot.	ft.
3 ft.	"	1 yard. . . .	yd.
5½ yd. or 16½ ft.	"	1 rod.	rd.
40 rd.	"	1 furlong. . .	fur.
8 fur. or 320 rd. or 1760 yd. or 5280 ft.	"	1 mile.	mi.

(*b*) TABLE OF GEOGRAPHIC LONG MEASURE.

3 geographic miles (mi.)	make	1 league.	lea.
60 geographic or 69⅙ English miles	"	1 degree of lat. or of longtitude on equator.	de or
360 degrees	"	1 great circle, g. c	

PROBLEMS.

1. How many inches in 3 feet? In 7 ft.? In 12 ft.?

2. How many feet in 24 inches? In 36 inches?

3. How many rods in 2 furlongs? In 5 fur.? In 3 fur.? In 2 miles? In 4 miles?

4. How many rods in 33 feet? In 66 feet? In 11 yards? In 22 yards? In 33 yards?

5. A cannon threw a ball 8000 yards; how many miles is that?

6. How many inches in 3 yards and 1 foot?

7. At 30 cents a foot, what will 13 yards 2 feet of wire railing cost? At 70 cents? At 40 cents?

LESSON XXXIV.

26. Cloth Measure is used by merchants in the sale of cloth, ribbons, laces, etc.

(*a*) TABLE.

2 sixteenths	make	1 eighth.
2 eighths	"	1 quarter.
2 quarters	"	1 half.
4 quarters or 2 halves or 36 inches	"	1 yard.

NOTE.—The old table of cloth measure is no longer used. By it the quarter is divided into 4 nails, each nail being 2¼ inches in length. Ells *are not used in the* United States.

PROBLEMS.

1. How many inches in 3 quarters? In 4 8ths? In 3 yards?

2. How many eighths in 5 quarters? In 3 yards? In 4 quarters?

3. How many quarters in 5 yards? In 6 yards? In 3 yards?

4. How many eighths in 12 16ths? In 16 quarters? In 5 qr.?

5. What will 3 sixteenths of cloth cost at 36 cents a qr.?

6. What is the cost of 3 eighths of ribbon at 40 cents a yard? At 60 cents? At 16 cents? At 8 cents?

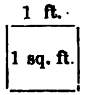
7. How many quarters in 2 yd. 2 qr.? In 3 yd. 2 qr. 3 8ths? In 5 yd.?

8. At 5 cents a quarter, what will 3 yd. 2 qr. of braid cost? At 9 cents? At 11 cents?

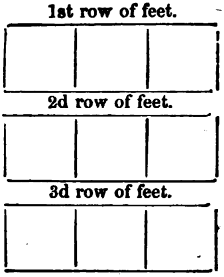
LESSON XXXV.

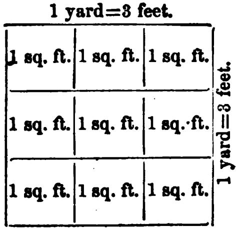
27. Square Measure is used to calculate areas, as of land, lumber, painting, paving, &c.

(*a*.) A **square** is a figure having four equal sides and four right angles or corners; a square foot is a square 1 foot in length *and 1 foot in* breadth, thus:



ILLUSTRATIONS.





FORM.—*Since in one row there are* 3 *square feet, in* 3 *rows there are* 3 *times* 3 *square feet, which is* 9 *square feet; hence in* 1 *sq. yd. there are* 9 *sq. ft.*

(*a*) TABLE.

144 square inches (sq. in.)	make	1 square foot, sq. ft.
9 sq. ft.	"	1 square yard, sq. yd.
30¼ sq. yd.	"	1 square rod, sq. rd.
40 sq. rds.	"	1 rood* . . . R.
4 R.	"	1 acre . . . . A.
640 A.	"	1 square mile, sq. mi.

PROBLEMS.

1. How many square feet in 5 square yards? In 7 square yards? In 11 sq. yd.? In 15 sq. yards?

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* A rood then is a quarter of an acre.

2. How many square yards in 36 sq. feet? In 108 sq. ft.? In 162 sq. ft.?

3. How many rods in 3 roods? In 2 quarters of an acre? In 3 quarters of an acre? In 2 acres? In 8 acres?

4. How many acres in 320 sq. rods? In 80 sq. rods? In 11 roods? In 800 rds.?

5. If one acre or 4 roods of land costs $28, what will 1 rood cost? 2 roods? 17 roods? 3 roods?

LESSON XXXVI.

28. Cubic Measure is used in estimating the contents of solids; as wood, stone, the capacity of cisterns, &c.

(*a*). A cube is a solid, having six equal square sides. Thus, a cubic foot is a body 1 foot in length, 1 foot in breadth, and 1 foot in thickness.

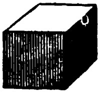
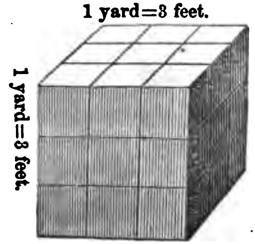
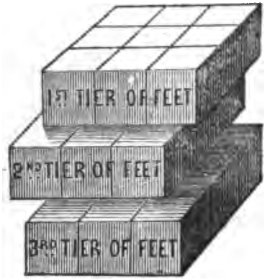


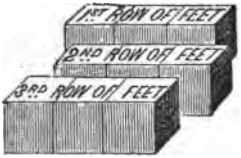
TABLE.

1728 cubic inches (cu. in.)	make	1 cubic foot, cu. ft.
27 " feet	"	1 cubic yard, cu. yd.
24¾ " " or 99 qr. cu. ft.	"	1 perch of stone, Pch.
16 cu. ft.	"	1 cord foot. c. ft.
8 c. ft. or 128 cu. ft.	"	1 cord of wood, C.

ILLUSTRATION.


1 yard=3 feet.
1 yard=3 feet.





1 C.FT.
1 C.FT.
1 C.FT.

(b.) Anal. Steps.—1. *Find the number of rows of cu. ft. in* 1 *cu. yd.*

Form.—Since there are 3 *rows* of cubic feet in 1 *tier* of feet, in 3 *tiers* there are 3 times 3 *rows*, which is 9 *rows*.

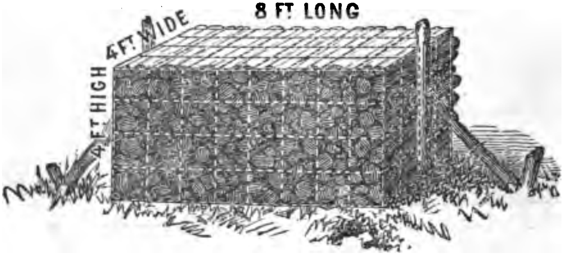
2. *Find the number of cubic feet in* 1 *cu. yard.*

Form.—Since in 1 row there are 3 *cubic feet*, in 9 *rows* there are 9 times 3 cubic feet, which is 27 cubic feet.

Concl.—Therefore, in one cubic yard there are 27 cubic feet. Hence to find the solid contents of a body we have the following

(c.) **Rule.**—*Multiply the* LENGTH *by the* BREADTH, *and that product by the* THICKNESS.

(*d.*) A pile of wood 4 feet wide, and 4 feet high and 8 feet long contains 1 *cord:* and a *cord foot* is 1 foot in length of such a pile, thus:



PROBLEMS.

1. How many cubic feet in 2 cubic yards? In 4 cu. yds.? In 1728 cu. in.? 3456 cu. in.?

2. How many *quarter* cubic ft. in a perch of stone? In 2 perch of stone?

3. How many perch of stone in 99 qr. cu. ft.? In 198 qr. cu. ft.?

4. How many cubic feet in a pile of wood 7 feet long 2 feet wide and 3 feet high?

5. How many cords and feet in a pile of wood 13 feet long 3 feet high and 4 feet wide?

6. How many cords in 24 cord feet? In 56 cord feet? In 75 cord feet? In 150 cubic feet?

LESSON XXXVII.

29. Liquid or Wine Measure is used for **measuring *all kinds of*** liquids.

(*a*) TABLE.

4 gills (gi.)	make	1 pint.	pt.
2 pt.	“	1 quart. . . .	qt.
4 qt.	“	1 gallon. . .	gal.
31½ gal.*	“	1 barrel. . . .	bbl.
2 bbls. or 63 gal.*	“	1 hogshead .	hhd.

(*b*) MISCELLANEOUS.*

36 gallons	make	1 barrel of beer.
54 gallons or 1½ barrels	“	1 hogshead of beer.
42 gallons	“	1 tierce.
2 hogshead or 126 gallons	“	1 pipe or butt.
2 pipes	“	1 tun.

PROBLEMS.

1. How many gills in 4 pints? In 6 pints? In 7 pints? In 11 pints? In 10 pints? In 14 pints?

2. How many pints in 3 quarts? In 20 quarts? In 47 quarts? In 1 gallon? In 4 gallons? In 12 gallons?

3. How many gills in 3 pints? In 3 gallons? In 7 quarts? In 7 gallons? In 4 quarts? In 4 gallons?

4. How many gills in 2 gallons 2 quarts? In 2 gallons 2 pints? In 2 pints 2 gills? In 2 gallons 2 gills? 2 gallons 3 quarts? 2 qt. 2 pt. 2 gi.?

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*NOTE.—This measure was formerly in use in the U. S., but is now nearly or quite obsolete.

5. How many quart bottles can be filled from 10 gallons of vinegar? How many pint bottles? How many gill cups? How many gallon jugs?

LESSON XXXVIII.

30. Dry Measure is used for solids; as grain, fruit, ashes, &c.

(*a*) TABLE.

2 pints	make	1 quart. . .	qt.
8 qt.	"	1 peck. . . .	pk.
4 pk.	"	1 bushel. . .	bu.

NOTE.—5 gills wine measure make about 1 pint dry measure.

PROBLEMS.

1. How many pints in a peck? In 3 pecks? In 5 qts.? In 4 pecks 4 pints? In 4 pecks 4 qts.?

2. In 84 pints how many qt.? How many pecks? How many bushels?

3. In 80 qt. how many pints? How many pecks? How many bushels?

4. In 30 pecks how many bushels? How many pints? How many quarts?

5. At 6 cents a pint, what will be the cost of a peck of hickory nuts? Of 3 quarts? Of 1 bushel?

6. I bought a bushel of chestnuts at 10 cents a *pint; how much* was that a peck?

7. 2 pecks of cranberries cost 96 cents; what was that per pint? Per qt.? Per bushel? Per 3 pecks? Per 2 qts.?

8. I sold a bushel of chestnuts at 12 cents a quart; how much did I receive for them?

9. I bought a bushel of strawberries for $6.40, and sold them at 40 cents a quart: what did I gain?

10. I bought a barrel of apples, containing 3 bushels, and sold them at the rate of 15 cents for 2 quarts; how much did I receive for them?

11. A man bought a bushel of potatoes for $3, he sold them at the rate of 20 cts. for 2 quarts; did he make or lose, and how much? If sold at the rate of 2 quarts for 30 cts.?

LESSON XXXIX.

31. Time is the measure of duration.

(*a*) TABLE.

60 seconds (sec.)	make	1 minute. . .	m.
60 m.	“	1 hour. . . .	hr.
24 hr.	“	1 day.	da.
7 da.	“	1 week. . . .	wk.
4 weeks 2 days	“	1 month. . .	mo.
12 calendar months	“	1 year.	yr.
365 days	“	1 yr.	
100 years	“	1 century. . .	C.

(*b*) DIVISIONS OF THE YEAR.

Seasons.			
Winter,	1. January,	31 days.	Jan.
	2. February,	28 or 29*	Feb.
Spring,	3. March,	31	Mar.
	4. April,	30	Apr.
	5. May,	31	May.
Summer,	6. June,	30	Jun.
	7. July,	31	July.
	8. August,	31	Aug.
Autumn,	9. September,	30	Sept.
	10. October,	31	Oct.
	11. November,	30	Nov.
Winter,	12. December,	31	Dec.
		365 or 366 days.	

Thirty days hath September,
April, June, and November,
February* hath twenty-eight,
And thirty-one the others rate.

LESSON XL.

PROBLEMS.

1. How many seconds in 2 minutes? In 3 minutes? In 4 minutes? In 12 minutes?

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[*] *February* has twenty-nine days in every Leap-year.

2. How many minutes in 120 seconds? In 3 hours? In 240 seconds? In 5 hours?

3. How many days in 3 months? In 3 months 12 days? In 5 months 7 days?

4. How many hours from 6 A. M. to 11 A. M.? From 4 A. M. to 12 M.? From 9 A. M. to 3 P. M.? From 11 A. M. to 10 P. M.

5. How many minutes from 20 minutes past nine to ten minutes past ten? From 7 minutes past 8 to 25 minutes before 11? Between 13 minutes past 11 o'clock A. M. and 12 minutes past 2 o'clock P. M.?

(*c*.) TABLE OF PARTICULARS.

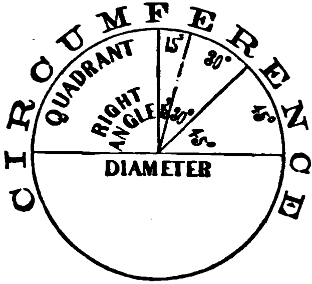
12 things	make	1 dozen.
12 dozen	"	1 gross.
12 gross	"	1 great gross.
20 things	"	1 score.
24 sheets of paper	"	1 quire.
20 quires "	"	1 ream.
56 pounds of corn	"	1 bushel.
60 " of wheat	"	1 bushel
196 " of flour	"	1 barrel.
200 pounds of beef, pork, or fish	"	1 barrel.

CIRCULAR OR ANGULAR MEASURE.

32. Circular or Angular measure is used to measure angles or the arcs of circles. It is used in astronomy, geography, navigation, and surveying, and for calculating *differences* of time.

(*a.*) TABLE.

60 seconds (″)	make	1 minute,	marked	′
60 minutes	"	1 degree,	"	°
30 degrees	"	1 sign,	"	sig.
90 degrees	"	1 quadrant, 1 right angle,	" "	quad. r. a.
4 quadrants or 12 signs	"	1 circumference or circle.	"	cir.



(*b.*) NOTES.—1. The greatest distance across a circle is called its *diameter*. The distance around it is called its *circumference*. Any part of the circumference is called an *arc*.

2. If any circumference, whether large or small, be divided into 360 equal arcs, each arc is called a degree. The degree is divided into 60 minutes, and the minute into 60 seconds. The length of a degree, minute, or second, depends on the size of the circle. If the size of the circle is increased or decreased, the length of the degree, minute, or second is also increased or decreased.

3. For the explanation of the principles of Longitude and Time see *Intermediate* or *Practical* Arithmetic, page 225.

TEST QUESTIONS FOR EXAMINATION.

ADDITION.—What is addition?[*] What is the result called in addition? Give the formula for adding 20 and 50. 60 and 70. 30 and 40. 90 and 80.

1.[†] Give the rule. What is the sum of 80 and 90. 30 and 70. 60 and 70. 20 and 30. 70 and 40.

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[*] For answers to these and similar questions the pupils are referred to the Practical or Intermediate Arithmetic.

[†] These figures refer to the paragraphs in the text of the book. In examination the teacher should test the pupils upon the practical problems, and see that they can readily give the appropriate formulas.

2. Give the formula for adding 87 and 50. 36 and 80. 90 and 67. Give the rule. What is the sum of 39 and 20? Of 37 and 80? Of 90 and 47? Of 60 and 39? What is the sum of 3+2+5+8+9+3+8+6+7+2+4+8? Of 3+5+6+7+8+9+3+4+7+6+4?

3. Give the formula for adding 63 and 25. 43 and 25. 98 and 71. Give the rule. What is the sum of 43 and 62? Of 27 and 91? Of 32 and 75? Of 93 and 74?

4. Give the formula for adding 36 and 78. 25 and 67. 97 and 39. Give the rule. What is the sum of 37 and 86? Of 35 and 67? Of 83 and 95. Of 24 and 39? Of 63 and 79? Of 39 and 78? Of 83 and 67? What is the sum of 637 and 463? Of 325 and 407? Of 397 and 763? Of 273 and 34?

5. Add from the blackboard the following numbers 36+24+86+37. Give the rule.

Subtraction.—What is subtraction? What is the result called? Which number is called the minuend? Which the subtrahend? Which the difference? Which the remainder? Which two terms are equal to the minuend? Which term subtracted from the minuend will give the remainder? Which term subtracted from the minuend will give the subtrahend?

6. Give the formula for subtracting 60 from 130. 90 from 180. 70 from 260. 50 from 520. Give the rule. What is the difference between 60 and 180? 50 and 170? 90 and 620? 40 and 520? 30 and 640? 90 and 370? 60 and 560? What is the difference between 30 and 121? Give the formula. Give the difference between the following numbers:—36 and 128. 73 and 144. 976—35. 437—62.

7. Give the formula for subtracting 39 from 137. Give the rule. What is the difference between the following numbers? 176 and 86. 374 and 98. 673 and 87.

8. What is the difference between 88 and 100? 73 and 100? Give the rule. 300 less 37 are how many? 300—63=what?

9. Give the formula for taking 37 from 463. Give the rule. What is the difference of 467 and 93? Of 378 and 27? Of 386 and 97? Of 567 and 38? Of 476 and 39?

Multiplication.—What is the result called in multiplication? Which number is the product? Which is the multiplicand? Which the multiplier? Which two terms multiplied together equals the product?

10. Give the formula for finding the product of 3 and 90. 6 and 70. Give the rule. 6 times 20 are how many?

11. Give the formula for multiplying 98 by 4. Give the rule. 6 times 89 are how many? 5 times 37 equals what? 6 times 37 equals what? How much longer will a cheese last 1 man than 8 men? 8 men will eat a cheese how much quicker than one man? How much more work can 9 men do than 1 man? A certain piece of work will last 1 man how much longer than 9 men?

12. How many are 13 times 17? Give the formula. Give the rule. How *many are 16 times 13?* 14 times 19? 16 times 19?

13. How many are 18 times 87? Give the formula. Give the rule. How many are 13 times 27? 16 times 86? 13 times 43? 12 times 83?

Division.—What is the number to be divided called? The number by which it is divided is called what? What is the result called? The product of what two numbers equals the dividend? What is meant by one-half of a thing or number? What is one ½ of an apple? How many in ½ of 6 apples? What is meant by 3 *fifths*? What is 3 *fifths* of one apple? What is 8 *fifths* of 10 apples? Of 15 apples? Of 25 peaches?

14. Give Form I. What is the result called in this formula?

15. Give Form II. What is the result called in this formula? What number is called the minuend? Which the subtrahend?

16. Give Form III. Which number is the multiplicand? Which the multiplier? Which the product? Which the result?

17. Give Form IV. Which number is the dividend? Which the divisor? Which the quotient? Which the result?

18. Give Form V. Which is the divisor in this formula? Which the dividend? Which the quotient? Which the result?

19. Give Form. VI. Which is the dividend? What rule does the 6th formula illustrate? Ans.—*Comparison.*

20. What is U. S. Money? What is meant by currency? Repeat the table. How are dollars separated from cents and mills? How many dimes in $6? Give the formula. What are the denominations of U. S. Money?

21. Where is English money used? What are the denominations? What is the value of a sovereign? Repeat the table. Repeat the table in a reverse order.

22. What is the use of Troy Weight? What are the denominations? Repeat the table. Repeat the table in a reverse order.

23. What is the use of Avoirdupois Weight? What are the denominations? Repeat the table. Repeat the table in a reverse order.

24. What is the use of Apothecaries' Weight? What are the denominations? Repeat the table. Repeat the table in a reverse order.

25. What is the use of Long Measure? What are the denominations? Repeat the table. Repeat the table of Geographic Measure.

26. What is the use of Cloth Measure? Repeat the table. What are the denominations?

27. What is the use of Square Measure? What is a square? What is the difference between a square foot and a foot square? Between 2 square feet and 2 feet square? Between 5 square feet and 5 feet square? Give the formula for finding the number of square feet in a square yard. Repeat the table.

28. What is the use of Cubic Measure? What is a cube? Repeat the table. Give the formula for finding the number of cubic feet in a cubic yard. Give the rule. What are the dimensions of a cord of wood?

29. *What is the* use of Liquid Measure? Repeat the table. What are *the denominations?*

30. What is the use of Dry Measure? Repeat the table. Give the denominations.

31. What is time? How is time measured naturally? Artificially? Repeat the table. How is the year divided? Give the names of the months in their order. How many days has each? How many days has February in leap-year. Repeat the table of particulars.

32. What is the use of Circular or Angular Measure? Repeat the table. What is meant by circumference? By diameter? By arc? By right angle? By quadrant?

◆

To the Teacher—The preceding subjects are to be studied in connection with the corresponding subjects of the *Intermediate*, *Practical*, or *Commercial Arithmetics*, and the time occupied is to vary according to the time spent in the same subjects in written arithmetic. It is very important that the pupil at each step should be subject to a searching examination. The test questions are only to be used as an aid for this purpose. Particular attention should be given to the following points:

1. *Rapidity and accuracy in combining numbers at sight.*

2. *Promptness and accuracy in abstract mental operations.*

3. *Distinctness in stating the analytical steps of complex problems.*

4. *Familiarity with the application of the five arithmetical formulas.*

FRACTIONS.

LESSON XLI.

NOTATION.

$\frac{1}{2}$, called one *half.*
$\frac{1}{4}$ " one *fourth.*
$\frac{1}{5}$ " one *fifth.*
$\frac{1}{6}$ " one *sixth.*
$\frac{3}{4}$, called 3 *fourths.*
$\frac{2}{5}$ " 2 *fifths.*
$\frac{3}{7}$ " 3 *sevenths.*
$\frac{5}{9}$ " 5 *ninths.*

33. **Fractions** are expressed by a numerator and a denominator.

34. **The Denominator** shows how many equal parts the unit is divided into, and gives the name to the fraction.

(*a.*) It may be written at the right of the numerator in words; as, 4 *tenths*, 9 *sevenths*, 8 *seventeenths*, or under the numerator, thus: $\frac{3}{7}$ numerator. denominator.

35. **The Numerator** shows how many there are of those equal parts.

(*a.*) It may be written either at the left of the denominator, as 3 *tenths*, 4 *sevenths*, or over the denominator, thus: $\frac{3}{10}$ numerator. denominator.

36. **The Numerator and Denominator** taken *together are called* the *terms* of a fraction.

1. When an apple is divided into 3 equal parts, what are the parts called? When divided into 4 equal parts? When into 7?

2. When an orange is divided into 8 equal parts, what are the parts called? When divided into 4 parts? 11 parts? 9 parts? 6 parts?

3. When a peach is divided into 2 equal parts, what is each one of the parts called? When divided into 5 equal parts? Into 3 equal parts? Into 7 equal parts? Into 11 equal parts?

4. When a melon is divided into 5 equal parts, what is one of the parts called? What are 2 parts called? 3 parts? 5 parts?

5. When a pie is cut into 8 equal parts, what are two parts called? 6 parts? 3 parts? 1 part? 7 parts? 4 parts?

6. What is meant by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{6}$, &c.? ANS.—*By one-half is meant* ONE *of the* TWO *equal parts into which a thing or number is divided; one-third is* ONE *of the* THREE *equal parts; one-fifth is* ONE *of the* FIVE *equal parts; &c.*

7. What is meant by $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{9}$, &c.?

ANS.—*By two-thirds is meant* TWO *of the three equal parts into which a unit or number of units is divided; the term three-fifths means* THREE *of the* FIVE *equal parts into which a unit or number of units is divided, &c.*

NOTE.—When no definite number of units is expressed, *one unit is always understood.*

8. When a melon is divided into six equal parts,

what is *one* of the parts called? ANS.—*One*-SIXTH *of* 1 *melon.*

9. When an orange is divided into six parts, what is 1 part called? What are 3 parts called? 4 parts?

10. When a pie is divided into 12 equal parts, what is 1 part called? What are 5 parts called? 7 parts? 8 parts?

LESSON XLII.

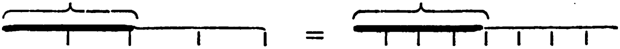
REDUCTION.

37. Principle.—If both numerator and denominator are multiplied by the same number, the value of the fraction will not be changed, because

The number of the parts increases in the same ratio as the size diminishes.

ILLUSTRATION.

$$\frac{2 \times 2}{4 \times 2} = \frac{4}{8}$$



Form a series* of fractions from each of the following primary fractions, by multiplying both numerator and denominator by each consecutive number from 1 to 20.

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* *To be read from slates or paper as a class exercise.*

MODEL.

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$, &c.
$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18}$, &c.
$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{12}{18}$, &c.

TABLE

Of all the primary fractions, the denominators of which are less than 13.

$\frac{1}{2}$	$\frac{1}{7}$	$\frac{1}{12}$	$\frac{2}{11}$	$\frac{3}{10}$	$\frac{4}{11}$	$\frac{5}{11}$	$\frac{7}{9}$	$\frac{8}{11}$
$\frac{1}{3}$	$\frac{1}{8}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{3}{11}$	$\frac{5}{6}$	$\frac{5}{12}$	$\frac{7}{10}$	$\frac{9}{10}$
$\frac{1}{4}$	$\frac{1}{9}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{7}$	$\frac{6}{7}$	$\frac{7}{11}$	$\frac{9}{11}$
$\frac{1}{5}$	$\frac{1}{10}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{5}{8}$	$\frac{6}{11}$	$\frac{7}{12}$	$\frac{10}{11}$
$\frac{1}{6}$	$\frac{1}{11}$	$\frac{2}{9}$	$\frac{3}{8}$	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{7}{8}$	$\frac{8}{9}$	$\frac{11}{12}$

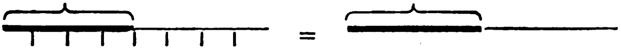
LESSON XLIII.

38. **Principle.**—If both numerator and denominator are divided by the same number, the value of the fraction will not be changed, because

The number of the parts diminishes in the same ratio as the size increases.

ILLUSTRATION.

$$\frac{4}{8} \div \frac{4}{4} = \frac{1}{2}$$



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BLACKBOARD EXERCISE.

MODEL.

$\frac{4}{8}$ $\frac{5}{15}$ $\frac{6}{24}$ $\frac{7}{35}$ $\frac{8}{48}$ $\frac{18}{27}$ $\frac{20}{50}$ $\frac{22}{77}$ $\frac{24}{108}$ $\frac{26}{143}$

NOTE FOR THE TEACHER.—Write on the blackboard exercises of fractions formed from the foregoing table of primary fractions.

RECITATION.—Let the teacher point, and the class name the primary fraction individually or in concert; thus, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. The exercise may be varied as follows:—

1. Commence at the left hand and pass toward the right.
2. Commence at the right hand and pass toward the left.
3. Commence at any place in the line and pass in either direction.
4. Point to any number, the class naming the result instantly.

LESSON XLIV.

39. **A Proper Fraction** is less than a unit; as $\frac{3}{4}$, $\frac{3}{7}$, &c.

40. **An Improper Fraction** is one equal to or greater than a unit, as $\frac{8}{8}$, $\frac{14}{7}$, &c.

41. **A Mixed Number** is a whole number added to a fraction: as $3+\frac{4}{7}$, $2+\frac{2}{3}$. The sign is usually omitted: as $7\frac{5}{8}$.

1. How many halves in 5 apples?

FORM.—*Since in* 1 *apple there are* 2 *halves, in* 5 *apples there are* 5 *times* 2 *halves, which is* 10 *halves, hence, in* 5 *apples there are* 10 *halves of an apple.*

2. How many fourths in 3 pears? 6 melons? 9 oranges? 8 pies? 12 lemons? 19 cakes?

3. How many *sevenths* in 3? In 4? In 6? In 5?

4. How many *fifths* in 6? In 13? In 12? In 9?

5. How many *sixths* in $3\frac{5}{6}$?

Form.—*Since in* 1 *unit there are* 6 *sixths, in* 3 *units there are three times* 6 *sixths, which added to* $\frac{5}{6}$ *equals* 23 *sixths; hence in* $3\frac{5}{6}$ *there are* $\frac{23}{6}$.

6. How many ninths in $3\frac{1}{9}$? In $4\frac{2}{9}$? In $6\frac{7}{9}$?

7. Write an improper fraction for each of the following mixed numbers.

MODEL.	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
$3\frac{1}{4}=\frac{13}{4}$	$4\frac{3}{7}=?$	$4\frac{3}{4}=?$	$6\frac{3}{4}=?$	$14\frac{3}{2}=?$	$5\frac{3}{7}=?$	$7\frac{2}{3}=?$
$4\frac{1}{7}=\frac{29}{7}$	$6\frac{7}{8}=?$	$8\frac{6}{7}=?$	$4\frac{5}{9}=?$	$11\frac{4}{7}=?$	$2\frac{5}{4}=?$	$8\frac{6}{7}=?$
$3\frac{1}{8}=\frac{25}{8}$	$5\frac{5}{7}=?$	$5\frac{8}{9}=?$	$7\frac{5}{8}=?$	$16\frac{3}{7}=?$	$6\frac{3}{17}=?$	$9\frac{5}{11}=?$
$5\frac{1}{9}=\frac{46}{9}$	$8\frac{3}{8}=?$	$4\frac{3}{7}=?$	$8\frac{6}{7}=?$	$14\frac{1}{8}=?$	$8\frac{6}{13}=?$	$8\frac{17}{14}=?$
$5\frac{4}{7}=\frac{39}{7}$	$7\frac{1}{8}=?$	$6\frac{6}{7}=?$	$13\frac{1}{4}=?$	$13\frac{5}{8}=?$	$4\frac{4}{11}=?$	$6\frac{13}{17}=?$
	$6\frac{8}{9}=?$	$5\frac{3}{7}=?$	$17\frac{2}{7}=?$	$6\frac{7}{8}=?$	$8\frac{3}{16}=?$	$5\frac{3}{7}=?$

8. How many apples in 12 halves of an apple?

Form.—*Since in* 2 *halves there is* 1 *apple, in* 12 *halves there are as many apples as* 2 *halves are contained times in* 12 *halves; which is* 6 *times; therefore in* 12 *halves there are* 6 *apples.*

9. How many oranges in 16 fourths? In 40 fifths?

10. How many units in 18 halves? In 24 quarters? In 32 eighths?

11. How many melons in 15 halves?

Form.—*Since in* 2 *halves there is* 1 *melon, in* 15 *halves there are as many melons as* 2 *halves are contained in* 15 *halves; which are* 7, *with one half remaining; hence, in* 15 *halves of a melon there are* $7\frac{1}{2}$ *melons.*

12. How many oranges in 19 quarters of an orange? In 20 quarters? In 11 quarters? In 48 quarters?

13. How many units in 63 fifths? In 64 sevenths?

14. How many units in 27 thirds? In $\frac{36}{7}$? In $\frac{17}{8}$? In $\frac{36}{7}$?

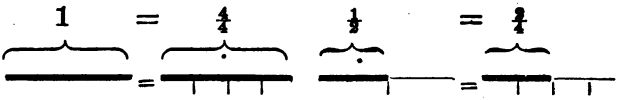
15. Write the corresponding mixed number for each of the following improper fractions.

MODEL.	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
$\frac{43}{7} = 6\frac{1}{7}$	$\frac{46}{7} = ?$	$\frac{47}{8}$	$\frac{56}{3}$	$\frac{49}{21}$	$\frac{46}{37}$	$\frac{59}{72}$	$\frac{43}{6}$
$\frac{27}{3} = 9$	$\frac{37}{2} = ?$	$\frac{38}{6}$	$\frac{72}{3}$	$\frac{73}{16}$	$\frac{97}{81}$	$\frac{41}{36}$	$\frac{94}{6}$
$\frac{63}{4} = 15\frac{3}{4}$	$\frac{46}{9} = ?$	$\frac{97}{6}$	$\frac{86}{7}$	$\frac{42}{13}$	$\frac{143}{67}$	$\frac{138}{71}$	$\frac{83}{21}$
$\frac{47}{4} = 11\frac{3}{4}$	$\frac{86}{8} = ?$	$\frac{97}{9}$	$\frac{96}{11}$	$\frac{72}{16}$	$\frac{43}{31}$	$\frac{68}{3}$	$\frac{47}{3}$
$\frac{68}{5} = 13\frac{3}{5}$	$\frac{47}{3} = ?$	$\frac{57}{3}$	$\frac{57}{9}$	$\frac{83}{18}$	$\frac{67}{32}$	$\frac{48}{21}$	$\frac{168}{82}$
$\frac{47}{13} = 9\frac{8}{13}$	$\frac{67}{9} = ?$	$\frac{67}{2}$	$\frac{68}{3}$	$\frac{89}{17}$	$\frac{47}{36}$	$\frac{67}{28}$	$\frac{46}{3}$

LESSON XLV.

1. How many quarters in a half?

ILLUSTRATION.



FORM.—*Since in one unit there are* 4 QUARTERS, *in one* HALF *of a unit there is* 1 HALF *of* 4 *quarters, which is* 2 QUARTERS; *hence in* $\frac{1}{2}$ *there are* $\frac{2}{4}$.

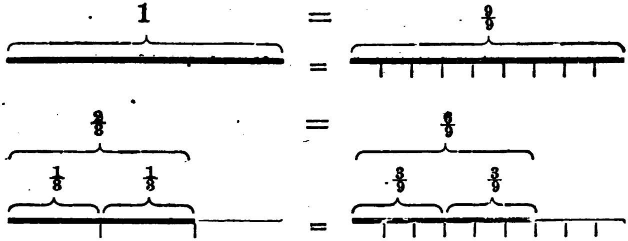
2. In $\frac{1}{3}$ how many sixths? How many twelfths? 18ths? 21sts.?

3. In $\frac{1}{4}$ how many eighths? How many 12ths? *16ths? 32ds?*

4. In $\frac{1}{6}$ how many 12ths? How many 18ths? 24ths? 48ths? 66ths?

5. How many *ninths* in $\frac{2}{3}$?

ILLUSTRATION.



FORM.—*Since in* 1 *unit there are* 9 *ninths, in* $\frac{1}{3}$ *of a unit there is* $\frac{1}{3}$ *of* 9 *ninths, which is* 3 *ninths; and in* 2 *thirds there are* 2 *times* 3 *ninths, which is* 6 *ninths; hence in* $\frac{2}{3}$ *there is* $\frac{6}{9}$.

6. In $\frac{3}{4}$ how many 12ths? 16ths? 36ths? 32ds?

7. In $\frac{4}{5}$ how many 25ths? 30ths? 70ths? 90ths?

8. How many 72ds in $\frac{1}{8}$? In $\frac{5}{6}$? In $\frac{5}{9}$? In $\frac{7}{12}$? In $\frac{5}{36}$?

9. How many 84ths in $\frac{3}{4}$? In $\frac{2}{3}$? In $\frac{2}{7}$? In $\frac{5}{6}$? In $\frac{5}{7}$?

10. How many 24ths in $\frac{2}{3}$? In $\frac{5}{8}$? In $\frac{5}{6}$? In $\frac{7}{12}$? In $\frac{5}{12}$?

11. How many 9ths in 7? In 11? In $\frac{1}{3}$? In $\frac{2}{3}$?

12. How many 24ths in 3? In $\frac{2}{3}$? In $3\frac{2}{3}$? In $6\frac{7}{12}$?

LESSON XLVI.

To find the simple or lowest terms of a fraction.

42. A **Fraction** is said to be expressed in its lowest terms when no number greater than ONE will divide both numerator and denominator without a remainder.

Change $\frac{6}{8}$ to its lowest terms.

ILLUSTRATION.

$$\frac{6}{8} \quad \frac{\div}{\div} \quad \frac{2}{2} \quad = \quad \frac{3}{4}$$

FORM.—2 is the greatest number that will exactly divide both terms of the fraction $\frac{6}{8}$; therefore the result $\frac{3}{4}$ is the lowest terms of the fraction.

(*a*). **Rule.**—*Divide the terms of the fraction by the greatest number that will divide them without a remainder.*

SLATE AND BLACKBOARD EXERCISES.

Express each of the following fractions in its lowest terms.

MODEL.	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
$\frac{3}{12} = \frac{1}{4}$	$\frac{12}{36} = ?$	$\frac{63}{81}$	$\frac{11}{77}$	$\frac{44}{88}$	$\frac{88}{99}$	$\frac{43}{86}$
$\frac{4}{12} = \frac{1}{3}$	$\frac{9}{15} = ?$	$\frac{63}{108}$	$\frac{21}{28}$	$\frac{42}{56}$	$\frac{46}{138}$	$\frac{57}{114}$
$\frac{12}{36} = \frac{1}{3}$	$\frac{5}{35} = ?$	$\frac{64}{72}$	$\frac{12}{84}$	$\frac{43}{86}$	$\frac{21}{189}$	$\frac{63}{189}$
$\frac{4}{18} = \frac{2}{9}$	$\frac{24}{44} = ?$	$\frac{132}{144}$	$\frac{35}{42}$	$\frac{72}{216}$	$\frac{30}{84}$	$\frac{12}{28}$
$\frac{5}{15} = \frac{1}{3}$	$\frac{40}{96} = ?$	$\frac{72}{90}$	$\frac{48}{36}$	$\frac{47}{94}$	$\frac{48}{72}$	$\frac{24}{44}$
$\frac{4}{24} = \frac{1}{6}$	$\frac{49}{77} = ?$	$\frac{24}{32}$	$\frac{35}{56}$	$\frac{83}{249}$	$\frac{69}{138}$	$\frac{24}{66}$

Recitation.—1. Require the above exercises to be copied on slate or on paper, completed as in the model, and read at recitation.

2. Require the numbers to be completed orally from the book without hesitation.

3. The teacher should question the class on the same.

1. What is the simplest form of $\frac{4}{8}$? Of $\frac{5}{10}$? Of $\frac{6}{14}$? Of $\frac{7}{21}$? Of $\frac{8}{10}$? Of $\frac{9}{12}$? Of $\frac{64}{72}$? Of $\frac{27}{36}$?

2. What are the lowest terms of $\frac{9}{12}$? Of $\frac{8}{16}$? Of $\frac{21}{28}$? Of $\frac{16}{48}$? Of $\frac{25}{30}$? Of $\frac{22}{77}$? Of $\frac{88}{99}$?

3. What are the lowest terms of $\frac{9}{18}$? $\frac{17}{32}$? $\frac{28}{42}$? $\frac{25}{75}$? $\frac{48}{72}$? $\frac{75}{100}$? $\frac{45}{90}$? $\frac{31}{62}$? $\frac{96}{108}$? $\frac{96}{120}$?

4. What are the lowest terms of $\frac{56}{24}$? $\frac{81}{27}$? $\frac{120}{180}$? $\frac{48}{96}$? $\frac{36}{84}$? $\frac{45}{90}$? $\frac{63}{27}$? $\frac{36}{72}$? $\frac{94}{188}$? $\frac{100}{700}$?

5. What are the lowest terms of $\frac{7}{14}$? $\frac{25}{100}$? $\frac{20}{100}$? $\frac{40}{100}$? $\frac{45}{100}$? $\frac{75}{100}$? $\frac{15}{100}$? $\frac{30}{100}$? $\frac{35}{100}$? $\frac{88}{100}$? $\frac{68}{100}$? $\frac{55}{100}$?

6. What are the lowest terms of $\frac{38}{100}$? $\frac{4}{100}$? $\frac{5}{100}$? $\frac{6}{100}$? $\frac{8}{100}$? $\frac{12}{100}$? $\frac{14}{100}$? $\frac{16}{100}$? $\frac{18}{100}$? $\frac{44}{100}$? $\frac{46}{100}$? $\frac{54}{100}$? $\frac{64}{100}$? $\frac{22}{100}$? $\frac{24}{100}$? $\frac{26}{100}$? $\frac{28}{100}$? $\frac{32}{100}$?

LESSON XLVII.

43. When **Fractions** are of the same denomination, they have a *common denominator.*

Change $\frac{2}{3}$ and $\frac{1}{4}$ to equal fractions of the same denomination.

Note.—Change $\frac{2}{3}$ and $\frac{1}{4}$ to the most convenient denomination.*

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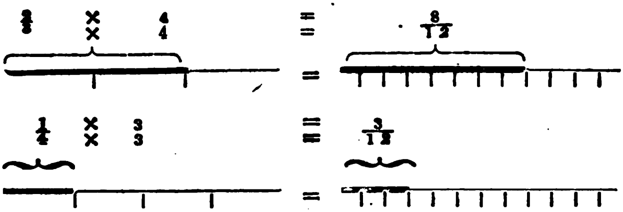
* Twelfths.

ILLUSTRATION.



Form.—Multiplying or dividing both terms of a fraction by the same number does not change its value; hence if the terms of $\frac{2}{3}$ are multiplied by 4, and the terms of $\frac{1}{4}$ are multiplied by 3, the resulting fractions $\frac{8}{12}$ and $\frac{3}{12}$ will be equal to the original fractions, $\frac{2}{3}$ and $\frac{1}{4}$, and have a common denominator.

(*d.*) **Rule.**—*Multiply or divide the terms of the fraction by any number that will make them of the same denomination.*

SLATE AND BLACKBOARD EXERCISES.

MODEL.*	(1.)	(2.)	(3.)	(4.)	(5.)
$\frac{3}{4}, \frac{2}{3}; \frac{9}{12}, \frac{8}{12}$	$\frac{2}{5}, \frac{4}{7};$?	$\frac{4}{5}, \frac{3}{7}$	$\frac{6}{7}, \frac{4}{8}$	$\frac{4}{7}, \frac{2}{3}$	$\frac{4}{15}, \frac{1}{2}$
$\frac{4}{7}, \frac{2}{3}; \frac{12}{21}, \frac{14}{21}$	$\frac{3}{8}, \frac{2}{7};$?	$\frac{3}{8}, \frac{4}{7}$	$\frac{2}{7}, \frac{13}{14}$	$\frac{4}{11}, \frac{13}{22}$	$\frac{5}{7}, \frac{3}{14}$
$\frac{5}{7}, \frac{3}{4}; \frac{20}{28}, \frac{21}{28}$	$\frac{4}{5}, \frac{3}{4};$?	$\frac{2}{6}, \frac{4}{5}$	$\frac{4}{7}, \frac{20}{21}$	$\frac{5}{11}, \frac{6}{7}$	$\frac{4}{5}, \frac{1}{15}$
$\frac{3}{4}, \frac{1}{8}; \frac{6}{8}, \frac{1}{8}$	$\frac{2}{5}, \frac{3}{7};$?	$\frac{1}{4}, \frac{3}{8}$	$\frac{2}{7}$ $\frac{25}{21}$	$\frac{4}{17}, \frac{11}{34}$	$\frac{2}{8}, \frac{3}{4}$
$\frac{5}{6}, \frac{1}{3}; \frac{5}{6}, \frac{2}{6}$	$\frac{4}{5}, \frac{2}{7};$?	$\frac{5}{7}, \frac{5}{14}$	$\frac{4}{7}, \frac{3}{4}$	$\frac{1}{6}$ $\frac{6}{11}$	$\frac{5}{7}, \frac{6}{7}$
$\frac{4}{7}, \frac{4}{5}; \frac{20}{35}, \frac{28}{35}$	$\frac{6}{7}, \frac{3}{8};$?	$\frac{5}{8}: \frac{3}{24}$	$\frac{5}{6}, \frac{13}{18}$	$\frac{4}{5}, \frac{3}{7}$	$\frac{2}{5}, \frac{3}{4}$

Recitation.—(See Lesson XLVI.)

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* **Notes.**—1. When one denominator can be divided exactly by the other it is necessary to multiply only one of the fractions; thus, $\frac{3}{4}, \frac{1}{8}$; $\frac{6}{8}, \frac{1}{8}$.

2. When both denominators cannot be divided exactly by any number, it is necessary only to write for the numerator of each fraction the product of each numerator and the denominator not its own, over the product of the denominators; *thus,* $\frac{1}{3}, \frac{3}{5}$; $\frac{5}{15}, \frac{9}{15}$.

6. Reduce $\frac{2}{3}$ and $\frac{3}{4}$ to a common denominator. Reduce $\frac{1}{4}$ and $\frac{2}{8}$. $\frac{1}{4}$ and $\frac{1}{5}$. $\frac{1}{7}$ and $\frac{1}{11}$. $\frac{1}{6}$ and $\frac{1}{9}$. $\frac{1}{7}$ and $\frac{1}{11}$. $\frac{1}{6}$ and $\frac{1}{8}$.

7. Reduce $\frac{2}{3}$ and $\frac{2}{4}$. $\frac{2}{7}$ and $\frac{2}{11}$. $\frac{3}{7}$ and $\frac{3}{11}$. $\frac{6}{7}$ and $\frac{5}{11}$. $\frac{4}{12}$ and $\frac{4}{13}$.

8. Reduce $\frac{1}{7}$ and $\frac{2}{11}$. $\frac{6}{7}$ and $\frac{4}{14}$. $\frac{5}{6}$ and $\frac{4}{12}$. $\frac{3}{6}$ and $\frac{2}{6}$. $\frac{4}{7}$ and $\frac{3}{8}$.

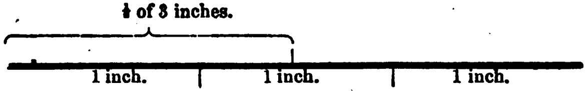
9. Reduce $\frac{2}{4}$ and $\frac{5}{12}$. $\frac{4}{7}$ and $\frac{3}{14}$. $\frac{2}{4}$ and $\frac{4}{3}$. $\frac{3}{2}$ and $\frac{5}{4}$. $\frac{4}{7}$ and $\frac{3}{12}$.

10. Reduce $\frac{1}{2}$ and $\frac{1}{4}$. $\frac{3}{7}$ and $\frac{5}{8}$. $\frac{4}{7}$ and $\frac{4}{8}$. $\frac{2}{4}$ and $\frac{5}{7}$. $\frac{4}{7}$ and $\frac{3}{8}$.

LESSON XLVIII.

1. What is $\frac{1}{2}$ of 3 inches? Ans.—*One of the two equal parts into which 3 inches is divided.*

ILLUSTRATION.



2. What is $\frac{1}{2}$ of 3 inches? Ans.—1½ *inch.*

3. What is $\frac{1}{3}$ of 1 inch? Ans.—$\frac{1}{3}$ *inch.*

4. What is $\frac{1}{4}$ of 1 inch? Ans.—$\frac{1}{4}$ *inch.*

5. What is $\frac{1}{7}$ of 1 pound? Ans.—$\frac{1}{7}$ *of a pound.*

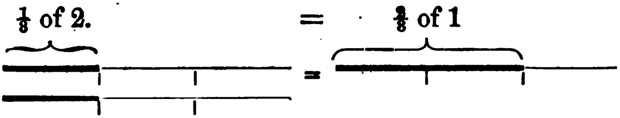
6. What is $\frac{1}{6}$ of 1 orange? What is $\frac{1}{9}$? $\frac{1}{12}$?

7. What is $\frac{2}{3}$ of 1 apple? Ans.—2 *times* $\frac{1}{3}$ *or* $\frac{2}{3}$.

8. What is $\frac{3}{4}$ of 1 gallon? $\frac{5}{9}$ of 1 barrel? $\frac{4}{5}$ of 1 yard? $\frac{7}{11}$ of 1 foot? $\frac{3}{7}$ of 1 cord?

9. What is $\frac{1}{3}$ of 2 apples? Ans.*—2 *times* $\frac{1}{3}$ *or* $\frac{2}{3}$ *of one apple.*

ILLUSTRATION.



Form.*—*Since* ONE-THIRD *of* 1 *apple is* $\frac{1}{3}$; ONE-THIRD *of* 2 *apples is* 2 *times* $\frac{1}{3}$, *which is* $\frac{2}{3}$.

10. What is $\frac{1}{3}$ of 3 apples? Ans.—1 *apple.*

11. What is $\frac{1}{3}$ of 4 apples? Ans.—1$\frac{1}{3}$ *apple.*

12. What is $\frac{1}{3}$ of 5 apples? Ans.—1$\frac{2}{3}$ *apple.*

13. What is $\frac{1}{3}$ of 7 apples? Ans.—2$\frac{1}{3}$ *apples.*

14. What is equal to $\frac{1}{7}$ of 6 apples?

Form.*—$\frac{1}{7}$ *of* 6 *apples must equal* 6 *times* $\frac{1}{7}$ *of one apple: that is,* $\frac{6}{7}$ *of one apple.*

15. What is $\frac{1}{11}$ of 7 pears? Of 3 pears? Of 9 pears?

16. What is $\frac{2}{11}$ of 5 pears? Of 7 apples? Of 9 oranges?

Form.—*Since* $\frac{1}{11}$ *of* 5 *pears equals* $\frac{5}{11}$ *of* 1 *pear, hence* $\frac{2}{11}$ *of* 5 *pears equals* 2 *times* $\frac{5}{11}$, *which is* $\frac{10}{11}$ *of* 1 *pear.*

17. What is $\frac{3}{7}$ of 8 apples? Of 6 apples? Of 4 apples?

Form.—$\frac{3}{7}$ *of* 8 *apples equals* 3 *times* $\frac{1}{7}$ *of* 8 *apples, which is* $\frac{24}{7}=3\frac{3}{7}$ *apples.*

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* To make these formulas and questions plain to the pupils, it is necessary *to illustrate* them by the use of visible objects.

18. What is $\frac{8}{9}$ of 7 oranges? Of 12 oranges? Of 16 lemons?

19. What is $\frac{5}{7}$ of 3? Of 6? Of 9? Of 8? Of 11?

LESSON XLIX.

To find the fractional part of a number.

1. What is equal to $\frac{1}{4}$ of a number? ANS.—*The number divided by* 4.

2. What is $\frac{1}{4}$ of 8? Of 17? Of 19? Of 27? Of 6?

3. What is equal to $\frac{3}{4}$ of a number? ANS.—3 *times the number divided by* 4.

4. What is $\frac{3}{4}$ of 7? Of 9? Of 8? Of 11? Of 13?

5. If a barrel of flour costs \$6, what will $\frac{5}{7}$ of a barrel cost?

OPERATION.—$\$6 \times 5 \div 7 = \frac{30}{7} = \$4\frac{2}{7}$.

FORM.*—$\frac{5}{7}$ *of a barrel will cost* $\frac{1}{7}$ *of* 5 *times as much as* 1 *barrel;* 1 *barrel costs* \$6, *therefore* $\frac{5}{7}$ *will cost* $\frac{1}{7}$ *of* 5 *times* \$6 *which is* $\$\frac{30}{7}$, *equal to* $\$4\frac{2}{7}$.

6. If a box of raisins cost \$5, what will $\frac{3}{7}$ of a box cost? $\frac{2}{8}$ of a box cost? $\frac{5}{9}$ of a box? $\frac{2}{7}$ of a box?

7. If a pound of tea is worth 90 cents, what

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[*] A contracted *formula* is better after the pupil learns to understand it.

will $\frac{1}{8}$ of a pound cost? $\frac{3}{8}$ of a pound? $\frac{5}{8}$ of a pound?

8. If a bushel of wheat is worth 95 cents, what is $\frac{1}{3}$ of a bushel worth? $\frac{3}{7}$ of a bushel? $\frac{2}{9}$ of a bushel? $\frac{4}{5}$ of a bushel?

SLATE AND BLACKBOARD EXERCISES.

Model Operation.—$\frac{3}{4}$ of $9 = 9 \times 3 \div 4 = \frac{27}{4} = 6\frac{3}{4}$ Ans.

MODEL.	(9.)	(10.)	(11.)	(12.)
$\frac{3}{4}$ of $9 = 6\frac{3}{4}$	$\frac{4}{5}$ of $2 =$?	$\frac{6}{7}$ of 8.	$\frac{5}{7}$ of 11.	$\frac{4}{7}$ of 9.
$\frac{5}{7}$ of $11 = 7\frac{6}{7}$	$\frac{7}{11}$ of $11 =$?	$\frac{2}{7}$ of 6.	$\frac{4}{7}$ of 10.	$\frac{5}{6}$ of 11.
$\frac{4}{5}$ of $3 = 2\frac{3}{5}$	$\frac{4}{5}$ of $13 =$?	$\frac{5}{6}$ of 8.	$\frac{5}{6}$ of 8.	$\frac{5}{7}$ of 16.
$\frac{6}{7}$ of $9 =$?	$\frac{3}{7}$ of $11 =$?	$\frac{3}{7}$ of 5.	$\frac{3}{7}$ of 17.	$\frac{2}{7}$ of 13.
$\frac{4}{5}$ of $3 =$?	$\frac{4}{9}$ of $3 =$?	$\frac{6}{7}$ of 9.	$\frac{4}{5}$ of 16.	$\frac{7}{8}$ of 12.
$\frac{6}{7}$ of $7 =$?	$\frac{5}{7}$ of $12 =$?	$\frac{2}{3}$ of 4.	$\frac{5}{7}$ of 11.	$\frac{4}{7}$ of 6.

Study and Recitation.—The above exercises should be copied and completed on slate or on paper, and read as a class exercise. It should be then completed at sight from the book; after which the class should be thoroughly examined upon it.

13. What will $\frac{5}{7}$ of a barrel of flour cost, if a barrel costs $11? If a barrel costs $6? $13? $15? $9?

14. What will $\frac{7}{9}$ of a pound of coffee cost if a pound costs 20 cts.? 30 cts.? 15 cts.? 11 cts.? 37 cts.? 25 cts.?

15. If a bushel of oats is worth 72 cts., what is the worth of $\frac{3}{7}$ of a bushel? Of $\frac{4}{5}$ of a bu.? $\frac{6}{7}$ of a bu.? $\frac{3}{8}$ of a bu.? $\frac{9}{11}$ of a bu.? $\frac{8}{9}$ of a bu.? $\frac{5}{8}$ of a bu.? $\frac{8}{11}$ of a bu.?

16. Mary had \$$\frac{3}{8}$ and Kate gave her \$$\frac{5}{8}$ and her brother William gave her \$$\frac{3}{8}$ more; how much had she in all?

17. I gave $\frac{3}{7}$ of an apple to Susan, and $\frac{1}{7}$ to Mary, how much did I give away?

18. $\frac{3}{8}+\frac{2}{8}$ are how many? $\frac{6}{8}$ less $\frac{2}{8}$ are how many? $\frac{5}{13}-\frac{2}{13}$=how many?

LESSON L.

44. ADDITION AND SUBTRACTION OF FRACTIONS.

To add or subtract fractions having 1 *for numerator.*

MODEL OPERATION.

$$\frac{1}{3}+\frac{1}{4}=\frac{4+3}{4\times3}=\frac{7}{12}\text{ Ans.}\qquad\frac{1}{3}-\frac{1}{4}=\frac{4-3}{4\times3}=\frac{1}{12}\text{ Ans.}$$

FORM.—1. $\frac{1}{3}=\frac{4}{12}$, $\frac{1}{4}=\frac{3}{12}$; $\frac{4}{12}+\frac{3}{12}=\frac{7}{12}$; $\frac{4}{12}-\frac{3}{12}=\frac{1}{12}$.

(*a.*) **Rule.**—I. *Write the sum of the denominators over their product.*

II.—*Write the difference of the denominators over their product.*

SLATE AND BLACKBOARD EXERCISES.

MODEL.*	(1.)	(2.)	(3.)	(4.)	(5.)
$\frac{1}{3}+\frac{1}{5}=\frac{8}{15}$	$\frac{1}{7}+\frac{1}{3}$	$\frac{1}{7}+\frac{1}{8}$	$\frac{1}{3}+\frac{1}{5}$	$5\frac{1}{5}+\frac{1}{3}$	$1\frac{1}{7}+\frac{1}{3}$
$\frac{1}{3}-\frac{1}{5}=\frac{2}{15}$	$\frac{1}{8}+\frac{1}{9}$	$\frac{1}{9}+\frac{1}{6}$	$\frac{1}{7}+\frac{1}{6}$	$8\frac{1}{6}+\frac{1}{3}$	$4\frac{1}{9}+\frac{1}{6}$
$\frac{1}{7}+\frac{1}{8}=\frac{15}{56}$	$\frac{1}{6}+\frac{1}{5}$	$\frac{1}{8}+\frac{1}{3}$	$\frac{1}{9}+\frac{1}{7}$	$7\frac{1}{4}+\frac{1}{8}$	$7\frac{1}{8}+\frac{1}{7}$
$\frac{1}{7}-\frac{1}{8}=\frac{1}{56}$	$\frac{1}{8}+\frac{1}{7}$	$\frac{1}{7}+\frac{1}{6}$	$\frac{1}{8}+\frac{1}{9}$	$1\frac{1}{3}+\frac{1}{6}$	$6\frac{1}{3}+1\frac{1}{6}$
$2\frac{1}{6}+\frac{1}{9}=2\frac{15}{54}=2\frac{5}{18}$	$\frac{1}{6}+\frac{1}{3}$	$\frac{1}{12}+\frac{1}{13}$	$\frac{1}{7}+\frac{1}{6}$	$2\frac{1}{7}+\frac{1}{3}$	$3\frac{1}{2}+2\frac{1}{3}$
$2\frac{1}{6}-\frac{1}{9}=2\frac{3}{54}=2\frac{1}{18}$	$\frac{1}{3}+\frac{1}{2}$	$\frac{1}{6}+\frac{1}{9}$	$\frac{1}{4}+\frac{1}{8}$	$6\frac{1}{8}+\frac{1}{7}$	$5\frac{1}{7}+\frac{1}{3}$

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* These exercises should be prepared on the slates like the model, and read at recitation as a class exercise; after which they should be solved mentally *from dictation.*

(6.)	(7.)	(8.)	(9.)	(10.)	(11.)
$\frac{1}{3}-\frac{1}{8}$	$\frac{1}{7}-\frac{1}{9}$	$\frac{1}{4}-\frac{1}{7}$	$\frac{1}{4}-\frac{1}{6}$	$2\frac{1}{5}-\frac{1}{7}$	$4\frac{1}{7}-1\frac{1}{8}$
$\frac{1}{9}-\frac{1}{12}$	$\frac{1}{6}-\frac{1}{11}$	$\frac{1}{5}-\frac{1}{8}$	$\frac{1}{7}+\frac{1}{5}$	$6\frac{1}{3}-2\frac{1}{8}$	$14\frac{1}{3}+6\frac{1}{8}$
$\frac{1}{7}-\frac{1}{18}$	$\frac{1}{15}-\frac{1}{21}$	$\frac{1}{7}-\frac{1}{12}$	$\frac{1}{8}-\frac{1}{18}$	$5\frac{1}{7}-3\frac{1}{18}$	$34\frac{1}{7}-30\frac{1}{8}$
$\frac{1}{5}-\frac{1}{6}$	$\frac{1}{8}-\frac{1}{12}$	$\frac{1}{6}+\frac{1}{11}$	$\frac{1}{5}+\frac{1}{6}$	$5\frac{1}{7}+4\frac{1}{11}$	$5\frac{1}{6}+2\frac{1}{7}$
$\frac{1}{4}-\frac{1}{8}$	$\frac{1}{7}-\frac{1}{13}$	$\frac{1}{5}-\frac{1}{7}$	$\frac{1}{6}+\frac{1}{7}$	$6\frac{1}{8}-3\frac{1}{11}$	$6\frac{1}{8}-2\frac{1}{9}$
$\frac{1}{6}-\frac{1}{8}$	$\frac{1}{3}-\frac{1}{17}$	$\frac{1}{6}-\frac{1}{8}$	$\frac{1}{8}+\frac{1}{12}$	$4\frac{1}{6}-8\frac{1}{8}$	$8\frac{1}{7}-1\frac{1}{12}$

12. I had $\frac{1}{4}$ of a dollar, and my brother gave me $\frac{1}{2}$ of a dollar more; how much had I then?

13. Austin paid $\$\frac{1}{8}$ for a pair of skates, and $\$\frac{1}{2}$ for a pair of mittens; how much did he give for both?

14. If a knife costs $\frac{1}{6}$ of a dollar, and a ball $\frac{1}{9}$ of a dollar; how much more does the knife cost than the ball?

15. Harry received $\$\frac{1}{11}$ from his father, and Thomas $\$\frac{1}{9}$ from his sister; how much more did one boy receive than the other?

16. If I sell $\frac{1}{6}$ of a dozen of eggs to one man, and $\frac{1}{8}$ of a dozen to another; how many dozen do I sell to both?

17. I sold $5\frac{1}{7}$ acres of land to one man and $3\frac{1}{8}$ acres to another; how many acres did I sell to both?

18. A man earned $\$13\frac{1}{8}$, and spent $\$6\frac{1}{12}$; how much had he left?

19. A farmer bought a sheep for $\$8\frac{1}{3}$: after he had paid $\$3\frac{1}{8}$, how much did he still owe?

20. William owned $\frac{1}{4}$ of a ship, and John $\frac{1}{8}$; *how much did* one own more than the other?

LESSON LI.

To add or subtract two fractions having like numerators.

MODEL OPERATION.

$\frac{2}{5}+\frac{2}{7}=\frac{7+5\times2}{7\times5}=\frac{24}{35}$ ANS. $\frac{2}{5}-\frac{2}{7}=\frac{7-5\times2}{7\times5}=\frac{4}{35}$ ANS.

FORM.—$\frac{2}{5}=\frac{14}{35}$, $\frac{2}{7}=\frac{10}{35}$; $\frac{14}{35}+\frac{10}{35}=\frac{24}{35}$; $\frac{14}{35}-\frac{10}{35}=\frac{4}{35}$.

(*b.*) **Rule.**—I. *Multiply the sum of the denominators by their common numerator, and write the product over the product of the denominators.*

II. *Multiply the difference of the denominators by their common numerator, and write the product over the product of the denominators.*

SLATE AND BLACKBOARD EXERCISES.

MODEL.	(1).	(2).	(3.)	(4.)
$\frac{2}{7}+\frac{2}{9}=\frac{32}{63}$	$\frac{4}{7}+\frac{4}{6}$	$\frac{3}{7}+\frac{3}{11}$	$\frac{2}{3}+\frac{4}{7}$	$3\frac{5}{7}+\frac{10}{11}$
$\frac{2}{7}+\frac{2}{8}=\frac{30}{56}=\frac{15}{28}$	$\frac{3}{7}+\frac{3}{9}$	$\frac{1}{2}+\frac{4}{9}$*	$\frac{6}{7}+\frac{6}{11}$	$2\frac{3}{7}+\frac{3}{4}$
$\frac{4}{5}+\frac{4}{5}=\frac{40}{25}=1\frac{3}{5}$	$\frac{4}{7}+\frac{4}{6}$	$\frac{6}{7}+\frac{3}{4}$	$\frac{8}{11}+\frac{4}{5}$	$4\frac{4}{5}+\frac{3}{8}$
$3\frac{2}{7}+4\frac{2}{9}=7\frac{32}{63}$	$\frac{5}{7}+\frac{5}{9}$	$\frac{5}{6}+\frac{5}{7}$	$\frac{8}{9}+\frac{2}{3}$	$3\frac{2}{7}+\frac{2}{9}$
$6\frac{3}{7}+2\frac{3}{7}=8\frac{42}{49}$	$\frac{3}{4}+\frac{3}{6}$	$\frac{1}{9}+\frac{3}{7}$	$\frac{5}{7}+\frac{5}{9}$	$4\frac{3}{4}+\frac{3}{7}$
$4\frac{3}{4}-2\frac{3}{5}=2\frac{3}{20}$	$\frac{4}{8}+\frac{4}{11}$	$\frac{7}{11}+\frac{7}{13}$	$\frac{6}{8}+\frac{6}{9}$	$7\frac{4}{5}+\frac{4}{7}$

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* Make the numerators alike, when it can be done, by multiplying the *terms of either or both* fractions by any number.

(5.)	(6.)	(7.)	(8.)	(9.)	(10.)
$\frac{2}{3}-\frac{3}{4}$	$\frac{4}{7}+\frac{4}{11}$	$\frac{5}{8}+\frac{5}{11}$	$3\frac{3}{7}-\frac{3}{8}$	$14\frac{2}{6}-3\frac{2}{11}$	$13\frac{2}{7}+\frac{2}{9}$
$\frac{3}{8}-\frac{3}{13}$	$\frac{5}{9}-\frac{5}{13}$	$\frac{5}{13}+\frac{5}{17}$	$4\frac{6}{7}-\frac{6}{8}$	$18\frac{3}{7}+2\frac{3}{11}$	$14\frac{3}{7}-\frac{3}{8}$
$\frac{4}{7}+\frac{4}{11}$	$\frac{5}{11}-\frac{5}{17}$	$\frac{6}{7}+\frac{6}{13}$	$15\frac{3}{4}-\frac{3}{7}$	$42\frac{3}{4}-20\frac{3}{5}$	$40\frac{4}{9}-\frac{4}{13}$
$\frac{5}{3}-\frac{5}{13}$	$\frac{3}{4}-\frac{3}{11}$	$\frac{4}{9}-\frac{4}{11}$	$15\frac{4}{9}-\frac{4}{11}$	$47\frac{3}{11}+40\frac{3}{13}$	$60\frac{3}{13}+\frac{6}{23}$
$\frac{9}{13}-\frac{9}{20}$	$\frac{4}{7}-\frac{4}{13}$	$\frac{8}{11}+\frac{8}{13}$	$3\frac{6}{7}+\frac{6}{8}$	$67\frac{2}{3}+60\frac{3}{7}$	$43\frac{6}{13}-\frac{6}{15}$
$\frac{6}{7}-\frac{6}{8}$	$\frac{6}{11}-\frac{6}{17}$	$\frac{6}{15}-\frac{6}{20}$	$4\frac{3}{7}+\frac{3}{11}$	$41\frac{3}{7}-40\frac{3}{13}$	$20\frac{3}{17}-\frac{3}{19}$

11. A boy bought $3\frac{1}{8}$ pounds of raisins at one time and $2\frac{1}{6}$ at another time; how many did he buy in all?

12. Strader bought $5\frac{2}{7}$ lb. of sugar at one time, and $6\frac{2}{9}$ pounds at another; how much did he buy in all?

13. Wilson sold $26\frac{3}{7}$ lb. of butter, and had $47\frac{3}{11}$ lb. left; how much had he in all?

14. Kate had $\$4\frac{5}{6}$, she spent $\$1\frac{5}{7}$ for a pocket-book; how much did she have left?

15. A man had $3\frac{2}{7}$ gal. of vinegar; after selling $1\frac{1}{2}$ gal. how much had he remaining?

16. James has $40\frac{3}{8}$ pounds of sugar; John has $11\frac{3}{7}$ pounds; James has how many pounds more than John?

LESSON LII.

To add or subtract two fractions having unlike numerators.

MODEL OPERATION.

$$\frac{3}{4}+\frac{5}{9}=\frac{(3\times9)+(5\times4)}{4\times9}=1\tfrac{11}{36}\text{ Ans. }\frac{3}{4}-\frac{5}{9}=\frac{(3\times9)-(5\times4)}{4\times9}=\tfrac{7}{36}\text{ Ans}$$

Form.—$\frac{3}{4}=\frac{27}{36}$, $\frac{5}{9}=\frac{20}{36}$; $\frac{27}{36}+\frac{20}{36}=1\frac{11}{36}$; $\frac{27}{36}-\frac{20}{36}=\frac{7}{36}$.

(*c.*) **Rule.***—I. *Multiply each numerator by the denominator not its own, and write their sum over the products of the denominators.*

II. *Multiply each numerator by the denominator not its own, and write their difference over the products of the denominators.*

SLATE AND BLACKBOARD EXERCISES.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
$\frac{3}{4}+\frac{5}{6}$	$\frac{4}{5}+\frac{6}{7}$	$\frac{6}{11}+\frac{5}{11}$	$\frac{3}{5}-\frac{1}{3}$	$\frac{2}{3}-\frac{2}{7}$	$4\frac{2}{11}-3\frac{1}{17}$
$\frac{4}{7}+\frac{5}{9}$	$\frac{3}{8}+\frac{5}{9}$	$\frac{3}{7}+\frac{4}{11}$	$\frac{4}{7}-\frac{3}{8}$	$\frac{6}{7}-\frac{4}{15}$	$2\frac{4}{7}-\frac{5}{13}$
$\frac{6}{7}+\frac{3}{4}$	$\frac{6}{7}+\frac{8}{9}$	$1\frac{3}{9}+\frac{6}{31}$	$\frac{6}{11}-\frac{5}{18}$	$\frac{6}{11}-\frac{2}{13}$	$4\frac{3}{4}+\frac{6}{11}$
$\frac{4}{7}+\frac{6}{7}$	$\frac{3}{4}+\frac{6}{7}$	$\frac{4}{7}-\frac{3}{7}$	$\frac{4}{7}-\frac{3}{9}$	$\frac{4}{11}-\frac{4}{13}$	$5\frac{4}{11}+\frac{3}{16}$
$\frac{5}{9}+\frac{3}{4}$	$\frac{5}{7}+\frac{3}{4}$	$\frac{5}{9}+\frac{8}{11}$	$\frac{6}{11}-\frac{7}{13}$	$\frac{6}{11}-\frac{5}{12}$	$6\frac{5}{12}-\frac{4}{13}$
$\frac{6}{8}+\frac{3}{4}$	$\frac{2}{7}+\frac{7}{9}$	$\frac{4}{7}+\frac{1}{3}$	$\frac{4}{15}-\frac{3}{17}$	$\frac{4}{13}-\frac{3}{16}$	$7\frac{2}{11}-\frac{1}{8}$

7. What is the difference between $4\frac{3}{7}$ and $2\frac{2}{9}$?

8. What must be added to $4\frac{3}{7}$ to make $8\frac{8}{9}$?

9. A boy bought a sled for $\$2\frac{1}{7}$; but becoming sick of his bargain sold it for $\$\frac{1}{9}$ less than he gave for it; how much did he receive for it?

10. Kate bought apples for $3\frac{1}{8}$ cts., nuts for $5\frac{5}{6}$ cts., and an orange for $2\frac{1}{2}$ cts., how much did she give for all?

11. Anna bought 5 oranges for $5\frac{1}{2}$ cts., 3 apples for $4\frac{1}{4}$ cts., 4 pounds of almonds for $25\frac{1}{8}$ cts.; how much did she give for all?

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* NOTE.—These rules apply to the class of fractions whose numerators or denominators are not multiples of each other.

LESSON LIII.

To subtract fractions from an integer, or to subtract fractions when the fractional subtrahend is greater than the fractional minuend.

MODEL OPERATION.

(1.)	(2.)	(3.)
41	$3\frac{2}{3}=3\frac{8}{12}$	$3-\frac{3}{4}=2\frac{1}{4}$
$23\frac{5}{7}$	$\frac{3}{4}=\frac{9}{12}$	$2\frac{1}{4}+\frac{2}{3}=1\frac{7}{12}$ Ans.
$17\frac{2}{7}$ Ans.	$2\frac{11}{12}$ Ans.	

FORM.—(2.) $3\frac{2}{3}=3\frac{8}{12}$; $\frac{3}{4}=\frac{9}{12}$; $\frac{9}{12}$ from $\frac{11}{12}$ leaves $\frac{3}{12}$; $\frac{3}{12}$ and $\frac{8}{12}$ equals $\frac{11}{12}$; 1 unit from 3 units leave 2; hence $3\frac{2}{3}-\frac{3}{4}=2\frac{11}{12}$.

(*d.*) **Rule.**—(3.) *Subtract the fractional subtrahend from a unit, and to the remainder add the fractional minuend.*

SLATE AND BLACKBOARD EXERCISES.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
$5-\frac{4}{5}$	$4\frac{3}{7}-\frac{5}{6}$	$3\frac{4}{7}-\frac{2}{3}$	$4\frac{3}{7}-\frac{4}{5}$	$6\frac{7}{8}-\frac{4}{5}$	$5\frac{6}{7}-\frac{4}{7}$
$3-\frac{2}{3}$	$8\frac{2}{3}-\frac{4}{7}$	$6\frac{2}{3}+\frac{4}{7}$	$2\frac{6}{8}+\frac{3}{7}$	$3\frac{1}{8}-\frac{6}{7}$	$3\frac{7}{5}-\frac{6}{7}$
$4-\frac{6}{7}$	$6\frac{4}{5}-\frac{4}{3}$	$5\frac{3}{4}-\frac{6}{7}$	$4\frac{2}{3}-\frac{3}{5}$	$4\frac{4}{11}-\frac{3}{11}$	$4\frac{2}{7}+\frac{3}{4}$
$5-\frac{4}{7}$	$7\frac{1}{7}-\frac{1}{6}$	$8\frac{3}{5}-\frac{3}{4}$	$6\frac{4}{7}-\frac{4}{9}$	$5\frac{2}{3}-\frac{4}{5}$	$6\frac{1}{7}-\frac{4}{5}$
$6-\frac{6}{7}$	$6\frac{5}{9}-\frac{1}{3}$	$9\frac{6}{7}-\frac{6}{8}$	$6\frac{3}{5}+\frac{6}{7}$	$4\frac{3}{4}+\frac{6}{7}$	$6\frac{1}{3}-\frac{4}{7}$
$5-\frac{2}{3}$	$4\frac{4}{5}-\frac{4}{3}$	$4\frac{1}{3}-\frac{1}{2}$	$5\frac{3}{8}+\frac{4}{7}$	$5\frac{2}{7}-\frac{3}{4}$	$8\frac{1}{7}+\frac{1}{7}$

NOTE.—In the following exercises add or subtract those fractions *which are the most* conveniently united, and to or from the sum or difference, add or subtract the remaining fractions.

7. $\frac{3}{4}+\frac{2}{3}+\frac{3}{8}$
8. $\frac{4}{5}+\frac{3}{10}+\frac{1}{2}$
9. $\frac{1}{8}+\frac{1}{6}+\frac{3}{4}$
10. $\frac{3}{4}+\frac{5}{6}+\frac{3}{8}$
11. $\frac{3}{7}+\frac{11}{14}+\frac{3}{4}$
12. $\frac{4}{5}+\frac{3}{10}+\frac{1}{3}$
13. $\frac{4}{5}+\frac{13}{15}+\frac{2}{10}$
14. $\frac{4}{5}+\frac{3}{4}+\frac{1}{2}$
15. $\frac{3}{4}+\frac{1}{5}+\frac{3}{10}$
16. $\frac{4}{7}-\frac{3}{14}+\frac{1}{2}$
17. $1\frac{3}{4}-\frac{2}{3}+\frac{5}{12}$
18. $\frac{4}{5}-\frac{3}{10}+\frac{3}{4}$
19. $\frac{4}{7}-\frac{3}{14}+\frac{1}{2}$
20. $\frac{3}{4}-\frac{2}{3}+\frac{5}{12}$
21. $\frac{4}{5}+\frac{3}{10}-\frac{3}{4}$
22. $\frac{5}{14}+\frac{1}{7}-\frac{2}{7}$
23. $\frac{1}{6}+\frac{1}{3}-\frac{3}{12}$
24. $\frac{4}{5}+\frac{1}{10}-\frac{3}{5}$

25. A man bought cloth to the amount of \$3$\frac{1}{5}$, \$6$\frac{7}{10}$, and \$4$\frac{1}{2}$; to how much did it amount?

26. I gave \$3$\frac{1}{8}$ for a cap, \$6$\frac{5}{8}$ for a hat, and \$4$\frac{1}{4}$ for a pair of boots; how much did all cost?

27. What is the weight of 3 pigs, if the first weighs 60$\frac{1}{2}$ lb., the second 80$\frac{1}{5}$ lb., and the third 75$\frac{1}{4}$ lb.?

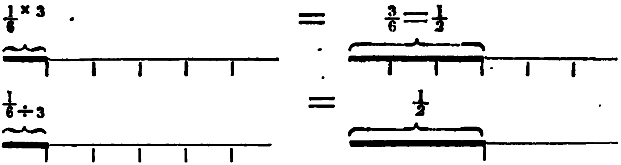
LESSON LIV.

45. MULTIPLICATION AND DIVISION OF FRACTIONS.

To multiply a fraction by an integral number.

What is 3 times $\frac{1}{6}$ of a line?

ILLUSTRATION.



FORM.—3 times $\frac{1}{6}$ are $\frac{3}{6}$, equal to $\frac{1}{2}$.

(*a.*) **Rule.***—MULTIPLY THE NUMERATOR or *divide the denominator.*

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* For principles and formulas see "Practical Arithmetic," page 235.

SLATE AND BLACKBOARD EXERCISES.

MODEL.	(1.)	(2.)	(3.)	(4.)	(5.)
	$\frac{4}{7}\times8$	$\frac{3}{7}\times4$	$\frac{4}{3}\times6$	$\frac{4}{7}\times5$	$1\frac{4}{7}\times6$
$\frac{4}{7}\times2=\frac{8}{7}=1\frac{1}{7}$	$\frac{3}{8}\times5$	$\frac{2}{7}\times8$	$\frac{5}{9}\times8$	$\frac{3}{7}\times8$	$3\frac{3}{7}\times8$
$\frac{3}{8}\times4=\frac{3}{2}=1\frac{1}{2}$	$\frac{6}{7}\times8$	$\frac{5}{9}\times6$	$\frac{3}{7}\times4$	$\frac{6}{7}\times3$	$4\frac{4}{9}\times3$
$\frac{6}{7}\times8=\frac{48}{7}=6\frac{6}{7}$.	$\frac{4}{5}\times6$	$\frac{2}{7}\times4$	$\frac{6}{7}\times3$	$\frac{5}{9}\times3$	$6\frac{7}{13}\times8$
$3\frac{1}{7}\times3=9\frac{12}{7}=10\frac{5}{7}$	$\frac{2}{7}\times7$	$\frac{3}{8}\times5$	$\frac{9}{13}\times5$	$\frac{7}{12}\times4$	$7\frac{5}{16}\times8$
$8\frac{3}{4}\times5=40\frac{15}{4}=43\frac{3}{4}$	$\frac{5}{7}\times3$	$\frac{4}{7}\times3$	$\frac{7}{11}\times8$	$\frac{7}{15}\times5$	$4\frac{13}{24}\times8$

6. If 1 pound of flour costs $\frac{3}{11}$ of a dime, what will 16 pounds cost ?

Form.—16 *pounds will cost* 16 *times as much as* 1 *pound; 1 pound costs* $\frac{3}{11}$ *of a dime, therefore* 16 *pounds cost* 16 *times* $\frac{3}{11}$ *of a dime, which is* $\frac{48}{11}$, *equal to* $4\frac{4}{11}$ *dimes.*

7. If 1 pound of tea costs $\frac{7}{8}$ of a dollar, what will 7 pounds cost ? 9 pounds ? 13 lb. ? 5 lb. ? 12 lb. ?

8. If 1 pound of sugar costs $7\frac{2}{3}$ cts., what will 8 lb. cost ? 9 lb. ? 6 lb. ? 10 lb. ? 11 lb. ?

9. If a barrel of flour is worth \$$9\frac{3}{4}$, what is 9 barrels worth ? 7 bbl. ? 8 bbl. ? 4 bbl. ? 12 bbl. ?

10. If 1 lb. of coffee is worth $37\frac{1}{2}$ cts., what are 5 pounds worth ? 3 lb. ? 7 lb. ? 6 lb. ?

11. What must I give for 5 lb. of cheese at $7\frac{3}{4}$ cts. per pound ? At $5\frac{3}{5}$ cts. ? At $7\frac{3}{5}$ cts. ? At $2\frac{1}{2}$ cts. ?

12. What must be given for 5 pounds of sugar, at $5\frac{3}{7}$ cts. per pound ? At $6\frac{1}{3}$ cts. ? At $7\frac{1}{2}$ cts. ? At $9\frac{2}{3}$ cts. ?

13. Bought 12 barrels of cider, at \$$3\frac{1}{3}$, and 3 barrels of vinegar, at \$$6\frac{1}{4}$ a barrel; what was the amount of the bill ?

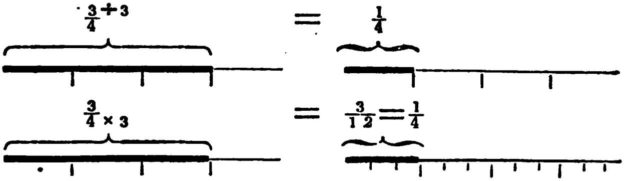
LESSON LV.

To divide a fraction by an integral number.

(1.) Divide $\frac{3}{4}$ by 3, or what is $\frac{1}{3}$ of $\frac{3}{4}$?

(2.) Divide $5\frac{3}{4}$ by 3, or what is $\frac{1}{3}$ of $5\frac{3}{4}$?

ILLUSTRATION.



(1.) **Form.**—1 third of $\frac{3}{4}$ is $\frac{1}{4}$.

(2.) **Form.**—3 is contained in 5 once with a remainder of 2; 2 is equal to $\frac{8}{4}$, and $\frac{8}{4}$ and $\frac{3}{4}$ are $\frac{11}{4}$, 1 third of $\frac{11}{4}$ is $\frac{11}{12}$: hence $5\frac{3}{4} \div 3 = 1\frac{11}{12}$.

(*b.*) **Rule.**—Divide the numerator or *multiply the denominator.*

SLATE AND BLACKBOARD EXERCISES.

MODEL	(1.)	(2.)	(3.)	(4.)	(5.)
$\frac{4}{7} \div 8 = \frac{4}{56} = \frac{1}{14}$	$\frac{5}{11} \div 3$	$\frac{4}{7} \div 4$	$\frac{4}{6} \div 8$	$8\frac{7}{9} \div 7$	$14\frac{3}{7} \div 2$
$\frac{6}{7} \div 3 = \frac{2}{7}$	$\frac{6}{7} \div 9$	$\frac{3}{7} \div 6$	$\frac{3}{7} \div 6$	$6\frac{7}{8} \div 3$	$16\frac{3}{4} \div 4$
$\frac{4}{7} \div 3 = \frac{4}{21}$	$\frac{4}{9} \div 7$	$\frac{5}{11} \div 6$	$\frac{3}{8} \div 6$	$4\frac{5}{8} \div 3$	$4\frac{5}{9} \div 2$
$3\frac{1}{7}$* $\div 4 = \frac{22}{28} = \frac{11}{14}$	$\frac{3}{7} \div 4$	$\frac{8}{17} \div 4$	$\frac{7}{12} \div 9$	$6\frac{7}{8} \div 8$	$13\frac{7}{11} \div 8$
$15\frac{3}{7} \div 7 = 2\frac{10}{49}$	$\frac{7}{8} \div 6$	$\frac{3}{16} \div 4$	$\frac{4}{7} \div 6$	$7\frac{4}{7} \div 3$	$3\frac{4}{7} \div 3$
$2\frac{6}{7} \div 4 = \frac{5}{7}$	$\frac{4}{11} \div 7$	$\frac{7}{18} \div 3$	$\frac{5}{9} \div 8$	$4\frac{6}{7} \div 4$	$20\frac{6}{7} \div 5$

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* When a mixed number is less than the divisor, reduce it to an improper fraction. In other cases divide separately. Pract. Arith., page 281.

6. If 5 oranges cost $\frac{3}{7}$ of a dime, what will one orange cost?

FORM.—*One orange will cost* $\frac{1}{5}$ *as much as* 5 *oranges;* 5 *oranges cost* $\frac{3}{7}$ *of a dime, therefore* 1 *orange will cost* 1 *fifth of* $\frac{3}{7}$ *of a dime, which is* $\frac{3}{35}$ *of a dime.*

7. If I give $\frac{3}{7}$ of a dime for 11 boxes of matches, what did 1 box cost?

8. What will be the cost of 1 box of pencils if 8 boxes cost \$$\frac{7}{8}$? \$$\frac{5}{9}$? \$$\frac{3}{4}$? \$$\frac{7}{11}$? \$$\frac{13}{16}$?

9. What will a lead-pencil cost, if 9 pencils cost \$$\frac{4}{5}$? \$$\frac{6}{7}$? \$$5\frac{3}{8}$? $18\frac{7}{11}$ cts.? $38\frac{4}{5}$ cts.?

10. If 13 oranges cost $10\frac{1}{2}$ cts., what will 4 cost?

ANAL. STEPS.—1. *Find the cost of* 1 *orange.*
2. *Find the cost of* 4 *oranges.*

11. If 5 lemons cost $10\frac{2}{3}$ cts., what will be the cost of 3? Of 7? Of 5? Of 9? Of 11?

12. If 3 pounds of butter cost $37\frac{1}{2}$ cts., what will 8 pounds cost?

13. If 5 pounds of rice cost $31\frac{1}{2}$ cts., what will 7 pounds cost? 8 lb.? 11 lb.? 3 lb.?

14. If 4 apples cost $\frac{5}{6}$ of a shilling, what will 1 apple cost? 6 apples? 7 apples? 9 apples?

15. If 8 eggs cost 25 cts., what will 3 cost? 5 cost? 9 cost?

16. If 3 pounds of sugar cost $21\frac{1}{3}$ cts., what will 8 pounds cost?

17. If 9 lb. of rice are worth $62\frac{1}{2}$ cts., what is the value of 5 pounds?

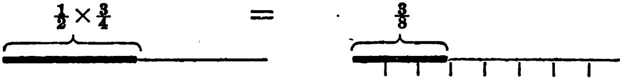
18. A boy gave $13\frac{1}{3}$ cts. for 9 apples; what was 3 worth at the same rate?

LESSON LVI.

To multiply by a fraction.

Multiply $\frac{1}{2}$ by $\frac{3}{4}$, or what is $\frac{3}{4}$ of $\frac{1}{2}$?

ILLUSTRATION.



FORM.*—$\frac{3}{4}$ of $\frac{1}{2}$ equals 3 times $\frac{1}{4}$ of $\frac{1}{2}$; $\frac{1}{4}$ of $\frac{1}{2}$ is $\frac{1}{8}$, and 3 times $\frac{1}{8}$ is $\frac{3}{8}$; hence $\frac{3}{4}$ of $\frac{1}{2}$ is $\frac{3}{8}$.

(*c.*) **Rule.**—I. MULTIPLY BY THE NUMERATOR, *and divide the product by the denominator*, or

II. *Write the product of the numerators over the product of the denominators.*

SLATE AND BLACKBOARD EXERCISES.

MODEL.	(1.)	(2.)	(3.)	(4.)	(5.)
$\frac{3}{4}\times\frac{2}{3}=\frac{6}{12}=\frac{1}{2}$	$\frac{2}{3}\times\frac{4}{5}$	$\frac{4}{5}\times\frac{3}{4}$	$\frac{4}{5}\times\frac{3}{4}$	$4\times\frac{3}{4}$	$6\frac{1}{3}\times\frac{2}{7}$
$\frac{4}{7}\times\frac{3}{8}=\frac{12}{56}=\frac{3}{14}$	$\frac{6}{7}\times\frac{3}{4}$	$\frac{6}{7}\times\frac{2}{3}$	$\frac{6}{7}\times\frac{5}{7}$	$6\times\frac{5}{8}$	$4\frac{1}{5}\times\frac{3}{7}$
$\frac{8}{9}\times\frac{5}{7}=\frac{40}{63}$	$\frac{5}{7}\times\frac{3}{4}$	$\frac{5}{7}\times\frac{6}{7}$	$\frac{3}{11}\times\frac{6}{7}$	$4\times\frac{3}{7}$	$6\frac{4}{9}\times\frac{5}{7}$
$6\times\frac{3}{7}=\frac{18}{7}=2\frac{4}{7}$	$\frac{6}{7}\times\frac{2}{3}$	$\frac{5}{8}\times\frac{3}{5}$	$\frac{5}{7}\times\frac{3}{7}$	$6\times3\frac{4}{7}$	$8\frac{6}{7}\times\frac{3}{7}$
$5\frac{3}{7}\times\frac{2}{3}=5\frac{3}{7}\times2\div3$†	$\frac{5}{6}\times\frac{3}{5}$	$\frac{7}{8}\times\frac{5}{9}$	$\frac{6}{7}\times\frac{5}{7}$	$2\times3\frac{3}{4}$	$4\frac{5}{8}\times\frac{2}{3}$
$4\frac{1}{5}\times\frac{3}{4}=3\frac{3}{20}$	$\frac{8}{9}\times\frac{6}{7}$	$\frac{5}{8}\times\frac{5}{7}$	$\frac{4}{9}\times\frac{3}{7}$	$6\times6\frac{2}{4}$	$16\frac{1}{5}\times\frac{1}{2}$

6. If 1 box of raisins costs $5, what will $\frac{3}{5}$ of a box cost?

FORM.—*If* 1 *box costs* $5, $\frac{3}{5}$ *of a box will cost* $\frac{1}{5}$ *of* 3 *times* $5, *which is* $3.

7. If a pound of flour is worth 9 cts., what are $\frac{3}{8}$ of a pound worth?

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[*] **NOTE.**—3 times $\frac{1}{4}$ of a thing or number is equal to $\frac{1}{4}$ of 3 times a thing or number; hence $\frac{1}{2}\div4\times3=\frac{1}{4}\times3\div4$.

[†] *NOTE.—Multiply the integral number and fraction separately, then add the products.*

8. At \$7 a barrel, what will $6\frac{2}{3}$ bbl. of flour cost?

FORM.—*$6\frac{2}{3}$ bbl. will cost $6\frac{2}{3}$ times as much as* 1 *barrel;* 1 *costs* \$7: *therefore $6\frac{2}{3}$ bbl. will cost $6\frac{2}{3}$ times* \$7, *which is* \$$42\frac{14}{3}$, *equal to* \$$46\frac{2}{3}$.

9. How much grain can be put into 15 bags if each bag holds $2\frac{1}{8}$ bushels?

10. What will be the cost of 12 pounds of rice at $6\frac{1}{4}$ cts. per pound?

11. If 1 pound of coffee costs \$$\frac{5}{8}$, what will $\frac{7}{11}$ lb. cost?

FORM.—*$\frac{7}{11}$ lb. will cost $\frac{1}{11}$ of* 7 *times as much as* 1 *pound costs;* 1 *pound costs* \$$\frac{5}{8}$: *therefore $\frac{7}{11}$ will cost $\frac{1}{11}$ of* 7 *times* \$$\frac{5}{8}$, *which is* \$$\frac{35}{88}$.

12. What will $\frac{3}{5}$ of a barrel of cider cost at \$$3\frac{1}{8}$ per barrel?

13. What will 17 pounds of honey cost, at $10\frac{3}{4}$ cts. per pound?

14. What will $12\frac{1}{8}$ pounds of honey cost at 20 cts. per pound?

15. If a dray-horse travels $\frac{6}{7}$ of a furlong in an hour, how far will he travel in $\frac{5}{8}$ of an hour?

16. At $\frac{3}{7}$ of a dime a pound, what will $15\frac{1}{2}$ pounds of nails cost?

17. Having \$50 I bought $8\frac{4}{9}$ tons of coal, at \$5 per ton; how much had I left?

18. If 1 pound of sugar costs 16 cts., what will $3\frac{5}{8}$ pounds cost?

19. What will $6\frac{3}{4}$ gallons of vinegar cost, at 14 cts. per gallon?

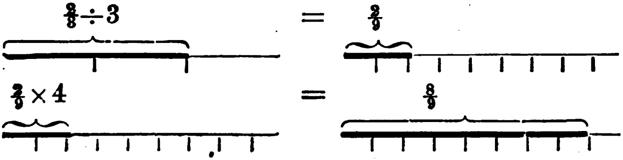
LESSON LVII.

To divide by a fraction.

MODEL OPERATION.

Divide $\frac{2}{3}$ by $\frac{3}{4}$. $\frac{2}{3}\div3\times4=\frac{8}{9}$ Ans.

ILLUSTRATION.



FORM.—One is contained in $\frac{2}{3}$ two-thirds of a time, hence 3 is contained in $\frac{2}{3}$, $\frac{1}{3}$ of $\frac{2}{3}$ of a time, and $\frac{1}{4}$ of 3, or $\frac{3}{4}$ is contained in $\frac{2}{3}$, 4 times $\frac{1}{3}$ of $\frac{2}{3}$, which is $\frac{8}{9}$.

(*d.*) **Rule.**—I. DIVIDE* BY THE NUMERATOR, *and multiply the quotient by the denominator*, or

II. *Invert the divisor, and write the product of the numerators over the product of the denominators.*

SLATE AND BLACKBOARD EXERCISES.

MODEL.	(1.)	(2.)	(3.)	(4.)	(5.)
$1\div\frac{3}{4}=\frac{4}{3}=1\frac{1}{3}$	$\frac{6}{7}\div\frac{3}{4}$	$1\div\frac{2}{3}$	$6\div\frac{3}{7}$	$1\frac{3}{4}\div\frac{2}{7}$	$6\frac{1}{7}\times\frac{3}{4}$
$\frac{4}{3}\div\frac{3}{7}=\frac{28}{9}=3\frac{1}{9}$	$\frac{2}{8}\div\frac{7}{9}$	$1\div\frac{6}{7}$	$4\div\frac{3}{7}$	$6\frac{1}{7}\div\frac{3}{8}$	$6\frac{1}{7}\div\frac{3}{4}$
$6\div\frac{4}{7}=\frac{42}{4}=10\frac{1}{2}$	$\frac{3}{5}\div\frac{4}{5}$	$1\div\frac{5}{8}$	$6\div\frac{2}{7}$	$5\frac{2}{7}\div\frac{4}{7}$	$4\frac{1}{3}\times\frac{5}{6}$
$3\frac{2}{3}\div\frac{6}{7}\dagger=21\frac{14}{3}\div6$	$\frac{6}{7}\div\frac{3}{7}$	$1\div\frac{3}{5}$	$4\div1\frac{2}{3}\ddagger$	$6\frac{3}{4}\div\frac{2}{7}$	$4\frac{1}{3}\div\frac{5}{6}$
$5\frac{2}{5}\div\frac{3}{4}=20\frac{3}{5}\div3$	$\frac{4}{6}\div\frac{3}{4}$	$1\div\frac{3}{7}$	$6\div2\frac{1}{3}$	$5\frac{2}{3}\times\frac{6}{7}$	$\frac{6}{7}\div1\frac{3}{4}$
$\frac{3}{4}\div1\frac{2}{3}\ddagger=\frac{9}{20}$	$\frac{2}{9}\div\frac{3}{4}$	$1\div\frac{7}{3}$	$5\div4\frac{1}{3}$	$3\frac{5}{7}\times\frac{3}{7}$	$\frac{6}{8}\times1\frac{3}{4}$

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* In practice it is generally more convenient to multiply before dividing.

† Divide the integral number and the fraction separately, then add the quotients.

‡ When the Divisor is a mixed number, always change it to an improper fraction.

6. Among how many beggars can $1 be distributed, if each receives $\frac{1}{5}$ of a dollar?

7. How many times will 1 contain $\frac{1}{4}$? $\frac{1}{6}$? $\frac{1}{8}$? $\frac{1}{11}$? $\frac{1}{20}$? $\frac{1}{17}$?

8. How many times is $\frac{1}{4}$ contained in 12? In 6? In 8? In 11?

FORM.—*Since* $\frac{1}{4}$ *is contained in* 1 FOUR *times, it is contained in* 12, *twelve times* 4 *which is* 48 *times; hence* $12\div\frac{1}{4}=48$.

9. At $\frac{1}{3}$ of a cent apiece, how many oranges can be bought for 12 cents? For 36 cts.? For 13 cts.?

10. 1 is how many times $\frac{2}{3}$? $\frac{3}{4}$? $\frac{7}{8}$? $\frac{6}{7}$? $\frac{4}{5}$? $\frac{8}{11}$?

FORM.—*Since* 1 *contains* $\frac{1}{3}$, 3 *times*, 1 *will contain* $\frac{2}{3}$, *one-half of* 3 *times, which is* $\frac{3}{2}=1\frac{1}{2}$.

11. 1 is how many times $\frac{3}{4}$? ANS.—$\frac{4}{3}=1\frac{1}{3}$. 1 is how many times $\frac{5}{8}$? $\frac{6}{7}$? $\frac{5}{9}$? $\frac{8}{11}$? $\frac{7}{11}$? $\frac{11}{7}$? $\frac{3}{11}$? $\frac{11}{3}$? $\frac{6}{7}$? $\frac{7}{6}$? $\frac{9}{11}$? $\frac{11}{9}$? $\frac{2}{3}$? $\frac{3}{2}$? $\frac{7}{9}$? $\frac{9}{7}$?

12. 2 are how many times $\frac{2}{3}$? $\frac{5}{7}$? $\frac{5}{8}$? $\frac{3}{11}$? $\frac{6}{11}$? $\frac{2}{7}$? $\frac{3}{5}$?

FORM.—*Since* 1 *contains* $\frac{2}{3}$, $\frac{3}{2}$ *of a time,* 2 *will contain* $\frac{2}{3}$, 2 *times* $\frac{3}{2}$ *which is* $\frac{6}{2}$ *equal to* 3.

13. $\frac{3}{4}$ is how many times $\frac{2}{3}$? $\frac{4}{5}$? $\frac{3}{8}$? $\frac{5}{6}$? $\frac{7}{9}$? $\frac{4}{11}$?

FORM.—Since 1 contains $\frac{2}{3}$, 3 halves of a time, $\frac{3}{4}$ of 1 will contain $\frac{2}{3}$, $\frac{3}{4}$ of $\frac{3}{2}$ which is $\frac{9}{8}=1\frac{1}{8}$, hence $\frac{3}{4}\div\frac{2}{3}=1\frac{1}{8}$.

14. How many pounds of sugar can be bought *for $3, at $$\frac{2}{7}$* per pound?

Form.—*$3 equals $$\frac{21}{7}$; since 1 pound costs $$\frac{2}{7}$, as many pounds can be bought for $$\frac{21}{7}$ as $$\frac{2}{7}$ are contained times in $$\frac{21}{7}$, which is $10\frac{1}{2}$ times, equal to $10\frac{1}{2}$ lb.*

15. How many yards of muslin can be bought for 57 cts., at $9\frac{1}{2}$* cts. per yd. ?

16. At $$\frac{1}{10}$ per quart, how many quarts of vinegar can be bought for $$\frac{1}{3}$?

Form.—*$1 will buy 10 times as much as $$\frac{1}{10}$; but $$\frac{1}{10}$ will buy a quart, hence $1 will buy 10 qt., and $$\frac{1}{3}$ will buy $\frac{1}{3}$ of 10 qt., or $3\frac{1}{3}$ qt.*

17. At $$2\frac{1}{4}$ per yard, how many yards of carpet can be purchased for $75 ?

18. If 13 oranges cost $10\frac{1}{2}$ cts., what will 4 cost ?

Anal. Steps.—1. *Find cost of 1 orange.*
2. *Find cost of 4 oranges.*

19. If 5 lemons cost $10\frac{2}{3}$ cts., what will 3 cost ? 7 ? 5 ? 9 ? 11 ?

20. If 3 pounds of butter cost $87\frac{1}{2}$ cts., what will 8 pounds cost ?

21. If 5 pounds of rice cost $31\frac{1}{2}$ cts., what will 7 pounds cost ? 8 pounds ? 11 pounds ? 3 pounds ?

22. If 4 apples cost $\frac{5}{6}$ of a shilling, what will 1 apple cost ? 6 apples ? 7 apples ? 9 apples ?

23. If $\frac{1}{2}$ of an apple costs $\frac{5}{6}$ of a cent, what will 1 apple cost ?

24. If $\frac{1}{3}$ of an apple costs $\frac{5}{6}$ of a cent, what will 1 apple cost ?

25. If $\frac{2}{3}$ of an apple costs $\frac{5}{6}$ of a cent, what will 1 apple cost ?

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* Reduce divisor to an improper fraction.

LESSON LVIII.

GENERAL ANALYSIS.

1. If 1 bushel of apples costs 5 shillings, what will $\frac{1}{7}$ of a bushel cost?

FORM.—*Since* 1 *bushel costs* 5 *shillings,* $\frac{1}{7}$ *of a bushel costs* $\frac{1}{7}$ *of* 5 *shillings, which is* $\frac{5}{7}$ *of a shilling.*

2. If a peck of corn is worth 30 cents, what is $\frac{1}{3}$ of a peck worth? $\frac{1}{5}$? $\frac{1}{9}$? $\frac{1}{11}$?

3. If a pound of sugar costs 19 cts., what will $\frac{1}{5}$ of a pound cost? $\frac{1}{4}$? $\frac{1}{9}$? $\frac{1}{6}$?

4. If a piece of meat weighs 17 lb-, what will $\frac{2}{3}$ of it weigh? $\frac{3}{4}$? $\frac{2}{5}$? $\frac{3}{7}$?

ANAL. STEPS.—1. *Find the weight of* $\frac{1}{3}$.
2. *Find the weight of* $\frac{2}{3}$.

FORM.—Since the whole weighs 17 lb., $\frac{2}{3}$ of it will weigh 2 times $\frac{1}{3}$ of 17 lb., which is $\frac{34}{3}$, equal to $11\frac{1}{3}$ lb.

5. A barrel of flour weighs 196 lb., how much will $\frac{3}{7}$ of a barrel weigh? $\frac{5}{8}$? $\frac{3}{4}$? $\frac{2}{7}$?

6. A yard of cloth is worth $3; what is $\frac{3}{8}$ of the same cloth worth? $\frac{5}{7}$? $\frac{3}{4}$? $\frac{2}{7}$?

7. A man's age is 47 years; his wife is $\frac{5}{8}$ as old: how old is his wife?

8. A merchant has $300 in the bank, and $\frac{3}{5}$ as much in his safe; how much has he in his safe?

9. 200 is 5 times a certain number; what is $\frac{4}{5}$ *of the number*? $\frac{3}{4}$? $\frac{5}{8}$? $\frac{3}{11}$?

10. 60 is 3 times a certain number; what is $\frac{3}{7}$ of the number? $\frac{2}{3}$? $\frac{3}{4}$? $\frac{1}{5}$?

11. Matthew had $\frac{5}{8}$ of a bushel of peaches; he sold $\frac{3}{8}$ of a bushel; how many had he left?

12. Helen paid \$$\frac{5}{6}$ for a geography, and \$$\frac{3}{4}$ for a reader; how much more did she give for the one than for the other?

Operation.—$\frac{5}{6}=\frac{10}{12}$, $\frac{3}{4}=\frac{9}{12}$; $\frac{10}{12}-\frac{9}{12}=\frac{1}{12}$ Ans.

Form.—*If the geography costs* \$$\frac{5}{6}$ *and the reader* \$$\frac{3}{4}$, *one must have cost as much more than the other as the difference between* $\frac{5}{6}$ *and* $\frac{3}{4}$, *which is* $\frac{1}{12}$.

13. Alexander had \$$\frac{7}{8}$; how much more must I give him, so that he may have \$$2\frac{1}{2}$?

14. Robert earns \$$7\frac{3}{4}$ a week; Joseph earns \$$1\frac{3}{8}$ less than Robert: how much does Joseph earn?

15. Horatio will be 21 years old in $3\frac{5}{12}$ years; how old is he now?

16. A boy sold $\frac{3}{4}$ of his nuts; what part had he remaining?

17. A gambler lost in play $\frac{1}{4}$ and $\frac{3}{8}$ of his money, what part of the whole did he lose? What part had he left?

18. A person spent \$$\frac{5}{8}$ for a book, \$$\frac{7}{8}$ for $\frac{1}{2}$ ream of paper, and had \$$\frac{3}{4}$ left; how much had he at first?

19. John wishes to pay \$$\frac{18}{25}$ for a pair of skates; he has $\frac{3}{5}$ of a dollar: how much more does he need?

20. By mistake a man paid \$$\frac{2}{3}$, which was \$$\frac{2}{10}$ too much; how much ought he to have paid?

21. A boy by mistake subtracted $\frac{7}{8}$ instead of $\frac{8}{7}$; was his answer too large or too small, and how much?

22. A man bought eggs for $\$\frac{8}{9}$, and sold them for $\$\frac{9}{10}$. Did he make or lose, and how much?

23. If a pound of tea costs $\$\frac{7}{8}$, what will 4 pounds cost?

FORM.—*Since* 1 *pound costs* $\$\frac{7}{8}$, 4 *pounds will cost* 4 *times* $\$\frac{7}{8}$, *which is* $\$\frac{7}{2}$, *equal to* $\$3\frac{1}{2}$.

24. If a pound of sugar costs $\$\frac{3}{8}$, what will 9 pounds cost?

25. If a gallon of molasses costs $\$\frac{9}{11}$, what will 6 gallons cost?

26. If $\$\frac{5}{8}$ will pay for a woman for working for one day; how much will pay for 7 days?

27. What will 5 barrels of flour cost at $\$6\frac{3}{4}$ a barrel?

28. If a barrel of beef is worth $\$15\frac{4}{9}$, what will 10 barrels be worth?

29. 1 orange costs $\$\frac{3}{18}$, what will 7 oranges cost?

30. 5 lb. of sugar cost $\$\frac{3}{5}$, what will 1 pound cost?

FORM.—1 *pound will cost* $\frac{1}{5}$ *as much as* 5 *pounds*; 5 *pound, cost* $\$\frac{3}{5}$, *therefore* 1 *pound will cost* $\frac{1}{5}$ *of* $\$\frac{3}{5}$, *which is* $\$\frac{3}{25}$.

31. 2 pounds of indigo cost $\$5\frac{2}{8}$;* how much is that a pound?

32. 6 eggs cost $25\frac{1}{2}$ cts.; how much is that apiece?

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* Divide the integral numbers and fractions separately.

33. 4 pounds of coffee cost 80¾ cts.; what will 1 pound cost?

34. 7 pounds of cheese are worth 87½ cts.; what is 1 pound worth?

35. 6 is ⅓ of what number?

Form.—*Since 6 is ⅓ of a number, ⅗ of it is 3 times 6, which is* 18.

36. 5 bushels are ⅐ of a load of apples; how many bushels in a load?

37. 5 is ⅛ of what number? $\frac{1}{11}$ of what number? $\frac{1}{13}$ of what number?

38. A clerk spent $60, which is ⅙ of his salary; what is his salary?

39. A farmer lost 20 sheep, which is ⅕ of his flock; how many sheep are in his flock?

40. 5 is ⅔ of what number?

Anal. Steps.—1. *Find ⅓ of the number.*
2. *Find ⅔ of the number.*

Form.—If 5 is ⅔ of a number, ⅓ of the number is ½ of 5, and ⅗, or the whole number, is 3 times ½ of 5, which is $\frac{15}{2}$, equal to 7½.

41. 7 is ¾ of what number? 6 is ⅝ of what number? 7 is ⅜ of what number?

42. $40 is $\frac{3}{7}$ of the value of my horse, what is its value?

43. $6 is $\frac{5}{7}$ of the cost of my coat, what is its cost?

44. 5 is $\frac{3}{7}$ of what number? 11 is $\frac{5}{7}$ of what number? 4 is $\frac{3}{7}$ of what number?

45. $\frac{2}{3}$ is $\frac{5}{7}$ of what number?

FORM.—*If* $\frac{2}{3}$ *is* $\frac{5}{7}$ *of the number,* $\frac{1}{7}$ *of the number is* $\frac{1}{5}$ *of* $\frac{2}{3}$, *and* $\frac{7}{7}$, *or the number, is* 7 *times* $\frac{1}{5}$ *of* $\frac{2}{3}$, *which is* $\frac{14}{15}$.

46. $\frac{5}{8}$ is $\frac{3}{7}$ of what number? $\frac{2}{7}$ is $\frac{4}{9}$ of what number? $\frac{3}{11}$ is $\frac{4}{7}$ of what number?

47. $\$\frac{3}{4}$ is $\frac{2}{7}$ of the cost of a pair of shoes; what did they cost?

48. $\frac{2}{8}$ of a yard is $\frac{2}{17}$ the height of my house; how many feet is it high?

49. $\$3\frac{1}{3}$ is $\frac{2}{9}$ of what I gave for a pair of pantaloons; what did they cost me?

50. I sold a sheep for $\$7\frac{1}{2}$, which was $\frac{3}{8}$ of what I gave for it; what did it cost me?

51. I sold a horse for $\$40\frac{1}{2}$ which was $\frac{2}{5}$ of what I gave for him; how much did I lose on him?

52. I gave $50 for a cow, which was $\frac{2}{5}$ as much as I sold her for; how much did I receive for her?

53. How many times is $\frac{1}{3}$ contained in 1? ANS.—3 *times.*

54. One is how many times $\frac{1}{7}$? ANS.—7 *times.*

55. At $\frac{1}{6}$ of a dollar a pound, how many pounds of sugar can be bought for $1?

FORM.—*Since there can be* 6 *times as much bought for* $1 *as for* $\$\frac{1}{6}$; *and since* $\frac{1}{6}$ *will buy* 1 *pound, therefore* $1 *will buy* 6 *times* 1 *pound or* 6 *pounds.*

56. At $\frac{1}{9}$ of a shilling apiece, how many apples can be bought for 1 shilling?

57. At $\frac{1}{11}$ of a cent apiece, how many hair-pins can be bought for a cent?

58. 1 is how many times $\frac{1}{8}$? 2 is how many times $\frac{1}{8}$? 6 is how many times $\frac{1}{8}$? 11 is how many times $\frac{1}{8}$?

59. At $\$\frac{1}{9}$ a pint, how many pints of vinegar can be bought for $2? For $3? For $4?

ANAL. STEP.—1. *Find how many can be bought for* $1.
2. *Find how many can be bought for* $2.

FORM.—$\$\frac{9}{9}$ or $1 will buy 9 times as much as $\$\frac{1}{9}$; $\$\frac{1}{9}$ will buy 1 pint, therefore $1 will buy 9 pints, and $2 will buy 2 times 9 pints, which are 18 pints.

60. How many peaches can I buy for 6 dimes, at $\frac{1}{3}$ of a dime apiece?

61. How many lemons can I buy for $5, if each lemon costs $\frac{1}{2}$ of a cent?

62. How many ninths of a yard can I cut from a piece containing 3 yards?

LESSON LIX.

MISCELLANEOUS PROBLEMS.

1. If I have $\$\frac{1}{2}$, and give away $\frac{1}{3}$ of it, how much shall I have left?

2. Into how many quarters can I cut 5 apples? Into how many sixths? Into how many twelfths?

3. What will 6 melons come to, if I can sell $\frac{1}{7}$ of 1 melon for 5 cts.? For 6 cts.? For 10 cts.?

4. How many 8ths in 1? In 3? In 12? In 20?

5. How many elevenths in 1? In 4? In 50? In 100?

6. How many 7ths in $8\frac{1}{7}$? In $3\frac{3}{7}$? In $6\frac{2}{7}$? In $4\frac{6}{7}$?

7. What must I give for a house, if $\frac{1}{5}$ of it is worth $400?

8. What is a mill worth if $\frac{1}{6}$ of it is worth $2400?

9. I have 6\frac{1}{5}$, how many dimes have I?

10. I have 24 quarters of an apple, how many whole apples have I?

11. In 30 ninths of a dollar, how many dollars are there?

12. How many units in $\frac{41}{7}$? In $\frac{24}{3}$? In $\frac{16}{8}$? In $\frac{24}{11}$?

13. Which is the greater, $\frac{1}{2}$ or $\frac{1}{4}$ of an apple? $\frac{1}{6}$ or $\frac{1}{7}$? $\frac{1}{9}$ or $\frac{1}{12}$? $\frac{1}{7}$ or $\frac{1}{9}$? $\frac{1}{5}$ or $\frac{1}{6}$? $\frac{1}{11}$ or $\frac{1}{10}$? $\frac{1}{8}$ or $\frac{1}{9}$?

14. Which is the greater, $\frac{2}{7}$ or $\frac{2}{8}$? $\frac{3}{7}$ or $\frac{3}{11}$? $\frac{4}{9}$ or $\frac{4}{8}$? $\frac{2}{7}$ or $\frac{2}{4}$? $\frac{11}{12}$ or $\frac{11}{13}$? $\frac{5}{7}$ or $\frac{5}{8}$? $\frac{13}{4}$ or $\frac{13}{3}$? $\frac{21}{3}$ or $\frac{21}{2}$?

15. Which is the greater, $\frac{1}{6}$ or $\frac{2}{6}$? $\frac{5}{9}$ or $\frac{4}{9}$? $\frac{8}{7}$ or $\frac{6}{7}$? $\frac{5}{9}$ or $\frac{3}{9}$?

16. Which fraction is of the greater denomination, $\frac{3}{7}$ or $\frac{3}{8}$? $\frac{4}{7}$ or $\frac{5}{8}$? $\frac{2}{7}$ or $\frac{3}{7}$? $\frac{3}{11}$ or $\frac{7}{9}$? $\frac{4}{7}$ or $\frac{5}{7}$? $\frac{2}{2}$ or $\frac{3}{2}$? $\frac{6}{11}$ or $\frac{11}{6}$?

17. What determines the denomination of a fraction? Which is the numerator? Which is the denominator?

18. What is $\frac{1}{8}$ of $25? $\frac{1}{9}$ of $17? $\frac{1}{5}$ of $20? $\frac{1}{11}$ of $37?

19. What is the number formed by adding $\frac{1}{7}$ and $\frac{2}{4}$?

20. What is the number formed by adding $\$\frac{3}{4}$ and $\$\frac{5}{6}$?

21. What is the sum of $\frac{3}{8}$ and of $\frac{2}{8}$ of a pound of sugar?

22. A boy has $\frac{3}{8}$ of a bushel of nuts, which is $\frac{1}{8}$ of a bushel less than his brother has; how many bushels has his brother?

23. $\frac{3}{7}$ is $\frac{2}{9}$ less than what number?

24. What number is larger by $\frac{3}{4}$ than $\frac{4}{7}$?

25. A man dug a ditch $\frac{3}{4}$ of a rod longer than his brother's door-yard, which was $\frac{5}{7}$ of a rod; how long was the ditch?

26. $\frac{3}{4}$ is the difference between $\frac{5}{8}$ and what larger number?

27. John had $\frac{3}{8}$ of a melon, which was the difference between $\frac{1}{6}$ and what part of a whole melon?

28. $\frac{3}{7}$ must be taken from what number to get $\frac{3}{4}$?

29. If you take $\frac{3}{8}$ from a certain number, $\frac{5}{7}$ will remain; what is the number?

30. A boy had $\frac{3}{4}$ of a pound of candy; he gave away $\frac{5}{8}$ of a pound; how much had he left?

31. What is the difference between $\frac{3}{4}$ and $\frac{3}{7}$?

32. $\$\frac{6}{11}$ is $\$\frac{6}{12}$ more than what part of a dollar?

33. What number is less by $\frac{2}{3}$ than $\frac{8}{9}$?

34. To what number must $\frac{3}{11}$ be added to get $\frac{3}{4}$?

35. $\frac{4}{7}$ is the difference between $\frac{10}{11}$ and what less number?

36. By adding $\frac{4}{5}$ to a certain number the sum is $\frac{6}{7}$; what is the number?

37. What is $\frac{1}{7}$ of 16? $\frac{1}{9}$ of 11? $\frac{1}{6}$ of 5?

38. What is $\frac{2}{3}$ of 11? Of 6? Of 12? Of 13?

39. What is $\frac{1}{5}$ of $\frac{1}{2}$? Of $\frac{3}{4}$? Of $\frac{5}{7}$? Of $\frac{6}{7}$?

40. What is $\frac{2}{7}$ of $\frac{2}{3}$? Of $\frac{6}{7}$? Of $\frac{4}{7}$? Of $\frac{3}{11}$?

41. What is $\frac{3}{5}$ of $2\frac{1}{3}$? Of $6\frac{1}{7}$? Of $5\frac{5}{7}$? Of $\frac{6}{7}$?

42. If 1 cake costs $\$\frac{7}{5}$, what will $\frac{1}{7}$ of it cost?

43. If 1 barrel of flour is worth $9, what will $\frac{3}{7}$ of a barrel be worth?

44. If a pound of figs costs $\frac{7}{8}$ of a dollar, what will $\frac{7}{8}$ of a pound be worth?

45. I gave $\frac{3}{4}$ of $16 for a barrel of fish, and $\frac{5}{12}$ of $24 for a tub of lard; how much did both cost?

46. What is $\frac{3}{4}$ of $14\frac{1}{2}$? Of $36\frac{1}{4}$? Of $24\frac{1}{5}$? Of $4\frac{1}{7}$?

47. 5 times $\frac{3}{4}$ are how many? 6 times $\frac{5}{7}$ equal what? $\frac{6}{11} \times 5$ equal what?

48. If 1 pig is worth $\$\frac{11}{3}$, what are 5 worth?

49. If $\$\frac{3}{7}$ will buy 1 dozen of eggs, what will 7 doz. cost?

50. Julia is $11\frac{2}{3}$ years old, Mary is twice as old, how old is Mary?

51. 3 times $16\frac{1}{2}$ are how many? $5\frac{2}{3} \times 4 =$ what? $6\frac{1}{7} \times 5 =$ what?

52. It costs $\$\frac{11}{3}$ to dig 1 rod of ditch, what must I give to have 7 rods dug?

53. 1 pound of rice costs $\$\frac{1}{7}$, what will ten pounds cost?

54. $80 is $\frac{1}{6}$ the cost of a house; what was given for it?

55. $\$\frac{3}{5}$ divided by 4 equals how much? $\frac{4}{5}\div 3=$ what?

56. 6 pounds of ginger are worth $\$\frac{5}{6}$? what is the value of 1 pound?

57. 2 doz. of eggs are worth $\$\frac{9}{11}$; what is the value of 1 dozen?

58. 1 doz. of knives are worth $\$3\frac{1}{3}$; what is that apiece?

59. 1 gallon of vinegar is worth $\$\frac{2}{3}$; how much is that per quart?

60. 1 yard of cloth is worth $\$3\frac{3}{4}$; how much is that a quarter?

LESSON LX.

1. At $\frac{1}{7}$ of a dollar a pound, how many pounds of cheese can be bought for $\$1$? For $\$2$? For $\$3\frac{1}{2}$?

2. If a boy earns $\frac{1}{8}$ of a dollar per day, how long will it take him to earn $\$1$? $\$3$? $\$4\frac{1}{2}$?

3. $\$1$ is how many times $\$\frac{2}{3}$?

Form.—*$\frac{1}{3}$ is contained in $\frac{3}{3}$ or* 1, 3 *times, and $\frac{2}{3}$ is contained in* 1, *$\frac{1}{2}$ of* 3 *times, which is $\frac{3}{2}$ times, equal to* $1\frac{1}{2}$.

4. 1 is how many times $\frac{3}{4}$? Ans.—$\frac{4}{3}=1\frac{1}{3}$?

5. 1 is how many times $\frac{5}{8}$? $\frac{6}{7}$? $\frac{3}{11}$? $\frac{11}{3}$? $\frac{2}{7}$?

6. $\$1$ is how many times $\$\frac{2}{7}$? $\$\frac{5}{8}$? $\$\frac{4}{7}$? $\$\frac{7}{4}$? $\$2\frac{2}{3}$?

7. How many pints of chestnuts can be bought for $\$1$, at $\$\frac{2}{11}$ per pint?

8. A man bought some beef for \$1; how many pounds did he get, at $\frac{3}{20}$ of a dollar per pound?

9. How many needles can be bought for a dime, if 1 needle is worth $\frac{2}{8}$ of a dime?

10. If 5 oranges cost \$1, what did 1 orange cost?

11. If $\frac{1}{5}$ of a cheese costs \$1$\frac{2}{3}$, what did the cheese cost?

FORM.—*If* $\frac{1}{5}$ *of a cheese costs* \$1$\frac{2}{3}$, *the cheese must have cost* 5 *times* \$1$\frac{2}{3}$, which is \$8$\frac{1}{3}$.

12. If $\frac{2}{5}$ of a cheese costs \$1$\frac{1}{2}$, what did the cheese cost?

FORM.—*If* $\frac{2}{5}$ *of a cheese costs* \$1$\frac{1}{2}$, *the cheese must have cost* $\frac{1}{2}$ *of* 5 *times* \$1$\frac{1}{2}$, *which is* $\frac{15}{4}$, *or* 3$\frac{3}{4}$ *dollars.*

N. B.—The pupil should be taught that the dividend is to be divided by the numerator of the divisor.

13. If 13$\frac{1}{2}$ lb. of sugar are worth \$1, what is the value of 1 lb.?

FORM.—13$\frac{1}{2}$ *equals* $\frac{27}{2}$; *if* $\frac{27}{2}$ *is worth* \$1, 1 *lb. is worth* $\frac{1}{27}$ *of* 2 *times* \$1, *which is* $\frac{2}{27}$ *of a dollar.*

14. If 9$\frac{1}{2}$ yards of lace are worth \$1, what is 1 yd. worth?

15. I gave \$1 for 11$\frac{3}{4}$ lb. of beef; how much was that a pound?

16. A lady gave \$1 for 2$\frac{2}{3}$ yd. of ribbon; how much was that a yard?

17. I gave \$1 for 2$\frac{3}{4}$ lb. of butter; what should I give for 5 pounds?

ANAL. STEPS.—1. *Find the cost of* 1 *lb.*
2. *Find the cost of* 5 *lb.*

FORM.—1. $2\frac{3}{4}=\frac{11}{4}$; if $\frac{11}{4}$ of a pound of butter costs $1, one pound will cost $\frac{1}{11}$ of 4 times $1, which is \$$\frac{4}{11}$.

2. If 1 lb. costs \$$\frac{4}{11}$, 5 pounds will cost 5 times \$$\frac{4}{11}$, which is \$$\frac{20}{11}$, equal to \$1$\frac{9}{11}$.

18. If $3\frac{2}{3}$ lb. of coffee cost $1, what must I give for 3 pounds?

19. If $6\frac{2}{7}$ lb. of cheese cost $1, what must I give for $\frac{2}{3}$ of a pound?

20. If $5\frac{3}{4}$ yd. of muslin are worth $1, what is the value of $\frac{2}{3}$ of a yard?

21. If $5\frac{5}{25}$ dozen of eggs cost $1, what should I give for $\frac{7}{12}$ of a dozen?

22. $3\frac{2}{3}$ gallons of vinegar are worth $1, what should be given for $5\frac{3}{4}$ gallons?

23. How many gallons of oil, at $\frac{8}{10}$ of a dollar a gallon, can be bought for $6\frac{7}{10}$.

FORM.—*If* 1 *gallon can be bought for* \$$\frac{8}{10}$, *for* \$$6\frac{7}{10}$ *or* \$$\frac{67}{10}$, *as many gallons can be bought as* \$$\frac{8}{10}$ *are contained in* \$$\frac{67}{10}$, *which are* $8\frac{3}{8}$ *times, hence* $8\frac{3}{8}$ *gallons.*

24. A man divided \$$2\frac{4}{5}$ among his children; to each he gave $\frac{7}{10}$ of a dollar; how many children are there?

25. How much flour can be bought for \$$\frac{3}{8}$, at $6 per barrel?

FORM.—*If* 1 *barrel can be bought for* $6, *as many barrels can be bought for* $\frac{3}{8}$ *of a dollar, as* $6 *is contained times in* \$$\frac{3}{8}$, *which is* $\frac{1}{16}$ *of a time; therefore* $\frac{1}{16}$ *of a barrel can be bought.*

26. At $7 a barrel, how much buckwheat can be bought for \$$\frac{5}{4}$? For \$$\frac{2}{3}$? For \$$\frac{6}{7}$?

27. At $11 per ton, how much coal can be bought for \$$\frac{3}{7}$? For \$$\frac{2}{3}$? For \$$\frac{3}{4}$?

28. At $\frac{2}{3}$ of a dollar a bag, how many bags of buckwheat can bought for \$$\frac{5}{7}$? For \$$\frac{3}{7}$? For \$$\frac{3}{4}$?

Anal. Steps.—1. *Find how much can be bought for* \$1.
2. *Find how much can be bought for* \$$\frac{5}{7}$.

29. If a horse can run $\frac{3}{8}$ of a mile in a minute, how many minutes would it take to run $\frac{5}{6}$ of a mile?

30. How many yards of cloth at $1\frac{1}{2}$ dollars a yard, can be bought for \$$2\frac{4}{5}$?

31. If a horse eats $\frac{3}{8}$ of a peck of oats per day, how long will $3\frac{1}{6}$ pecks last him?

32. If a miller takes for toll $\frac{6}{25}$ of all the rye that he grinds, how many bushels must he grind to receive $9\frac{2}{7}$ bushels?

33. 9 is $\frac{1}{4}$ of what number? $\frac{1}{3}$ of what number? $\frac{1}{7}$? $\frac{1}{5}$? $\frac{1}{11}$?

34. 7 is $\frac{1}{7}$ of what number? $\frac{2}{7}$ of what number? $\frac{3}{7}$? $\frac{2}{3}$? $\frac{4}{5}$?

35. Jane gave 11 cts. for a writing book, which was $\frac{3}{4}$ of her money; how much had she?

Form.—*If* 11 *cts. is* $\frac{3}{4}$ *of her money,* $\frac{1}{4}$ *of it must be* $\frac{1}{3}$ *of* 11 *cts. which is* $\frac{11}{3}$ *of a cent, and* $\frac{4}{4}$, *or the whole, must be* 4 *times* $\frac{11}{3}$ *of a cent, or* $\frac{44}{3}$ *of a cent, or* $14\frac{2}{3}$ *cts.*

36. A man bought a cow for \$30, which was $\frac{3}{5}$ of his money; how much had he?

37. A boy gave for some candy 2 cents, which was $\frac{2}{3}$ of his money; how much had he?

38. A girl gave 1 cent for some pins, which was $\frac{3}{12}$ of her money; how much had she at first? How much had she left?

39. A girl gave $\frac{1}{2}$ dime for a pencil, which was $\frac{4}{7}$ of her money, how much had she at first? How much had she left?

40. Kate was married at 22 years of age, which was $\frac{11}{17}$ of her husband's age, how old was her husband?

41. Two men built a wall; one man built 16 rods, which was $\frac{4}{5}$ of what the other built; how much did the other build? What was the entire length of the wall?

42. If $\frac{2}{3}$ of a yard of cloth costs \$3, what will $\frac{3}{4}$ of a yard cost?

ANAL. STEPS.—1. *Find cost of* 1 *yard.*
2. *Find cost of* $\frac{3}{4}$ *yard.*

FORM.—If $\frac{2}{3}$ yard costs \$3, 1 yard will cost $\frac{3}{2}$ of \$3, which is $\frac{9}{2}$; and $\frac{3}{4}$ yd. will cost 3 times $\frac{1}{4}$ of \$$\frac{9}{2}$, which is \$$\frac{27}{8}$, equal to \$3$\frac{3}{8}$.

43. If $\frac{3}{4}$ of a yard contains 27 inches, what is the length of $\frac{2}{3}$ of a yard?

44. \$15 is $\frac{3}{5}$ of the monthly rent of a house; how much is that per year?

45. If $\frac{1}{4}$ of a pound of ginger costs \$$\frac{7}{8}$, what will $\frac{2}{7}$ of a pound cost?

46. If $\frac{5}{7}$ of a yard of silk is worth \$2$\frac{5}{7}$, what is the value of $\frac{4}{3}$ of a yard?

47. Delphine saves $\frac{1}{5}$ of a dollar, and Harriet $\frac{1}{4}$ of a dollar in a week; how long at that rate will it take them to save \$$\frac{15}{20}$?

48. How many yards of cloth, at \$1$\frac{1}{2}$ a yard can be bought for \$2$\frac{4}{5}$?

LESSON LXI.

DENOMINATE FRACTIONS.

46. ENGLISH MONEY—*Table.*

1 pound = 20*s.* = 240*d.* = 960 far.
1 shilling = 12*d.* = 48 far.
1 penny = 4 far.

1. How many pence in $\frac{2}{7}$ of a pound?

Form.—*Since in £1 there are 240d., in $\frac{2}{7}$ of a pound there are 2 times $\frac{1}{7}$ of 240d., which is 68$\frac{4}{7}$d.*

2. How many farthings in $\frac{3}{5}$ of a shilling? In $\frac{1}{7}$ of a penny?

3. How many farthings in $\frac{2}{3}$ of a pound? In $\frac{5}{6}$ of a pound?

4. How many shillings in $\frac{3}{4}$ of a pound? In $\frac{5}{6}$ of 3 pounds?

47. TROY WEIGHT—*Table.*

1 lb. = 12 oz. = 240 pwt. = 5760 gr.
1 oz. = 20 " = 480 "
1 pwt. = 24 "

5. How many ounces of silver in $\frac{3}{7}$ of a pound?

6. What is $\frac{2}{7}$ of a pound of gold-dust worth at $10 per ounce?

Form.—1. *Since there are 12 oz. in 1 pound, in $\frac{2}{7}$ of a pound there are 2 times $\frac{1}{7}$ of 12 oz., which is $\frac{24}{7}$ oz., equal to 3$\frac{3}{7}$ oz.*

2. *If 1 oz. is worth $10, 3$\frac{3}{7}$ oz. are worth 3$\frac{3}{7}$ times $10, which is 30\frac{10}{7}$, equal to 31\frac{3}{7}$.*

(*a.*) APOTHECARIES WEIGHT.—*Table.*

1℔ = 12℥ = 96ʒ = 288℈ = 5760 gr.
1℥ = 8ʒ = 24℈ = 480 "
1ʒ = 3℈ = 60 "
1℈ = 20 "

7. What is the value of ⅜ of an ounce?

48. AVOIRDUPOIS WEIGHT—*Table.*

1 T. = 20 cwt. = 80 qr. = 2000 lb.
1 cwt. = 4 qr. = 100 "
1 lb. = 16 oz. = 256 dr.
1 oz. = 16 dr.

8. What must be given for ⅜ of a ton of coal, at 50 cents per cwt.?

9. I paid $12 for a ton of coal; how much was that per cwt.? How much was it per ⅜ cwt.?

FORM.—*If* 20 *cwt., or* 1 *ton, costs* $12, 1 *cwt. costs* $\frac{1}{20}$ *of* $12 *which is* $\$\frac{12}{20}=\$\frac{3}{5}$.

10. What will ⅜ of an ounce of opium cost at $5 per pound? At $4⅗? At $6⅕?

11. Bought ⅖ cwt. of flour for $1⅛; how much was that per pound?

ANAL. STEPS.—1. *Find cost of* 1 *cwt.*
2. *Find cost of* 1 *lb.*

12. I paid 12 cents for 5 pounds of flour; how much was that per cwt.?

13. Bought $\frac{3}{7}$ cwt. of sugar for $8; how much was that per pound?

14. A man bought $\frac{3}{8}$ of a pound of sugar for 5 cts.; what was that per cwt?

15. How many cents in $\frac{2}{5}$ of a dollar? In \$$\frac{1}{8}$? In \$$\frac{5}{7}$? In \$$\frac{6}{7}$.

FORM.—*Since in* \$1 *there are* 100 *cents, in* \$$\frac{2}{5}$ *there are* $\frac{2}{5}$ *of* 100 *cents, which is* $\frac{200}{5}$, *or* 40 *cts.*

16. I gave \$$\frac{3}{4}$ for $\frac{2}{7}$ of a pound of cinnamon; how many cents was that a pound?

17. 3 ounces of cloves cost \$$\frac{3}{11}$; how many cents is that per ounce?

49. LONG MEASURE—*Table.*

1 mile	=	320 rd.	=	1760 yd.	=	5280 ft.
1 rd.			=	$5\frac{1}{2}$ "	=	$16\frac{1}{2}$ "
1 yd.			=	3 ft.	=	36 in.
1 ft.					=	12 in.

18. What must be given for $\frac{3}{7}$ of a yard of cloth, if 6 inches of the cloth are worth 5 cents?

19. How many feet in $\frac{3}{4}$ of a yard? In $\frac{3}{4}$ of a rod?

20. How many inches in $\frac{2}{3}$ of a yard? In $\frac{5}{9}$ of a ft.?

50. CLOTH MEASURE—*Table.*

1 yd.	=	4 qr.	=	8 eighths	=	36 in.
1 qr.			=	2 "	=	9 in.
1 eighth					=	$4\frac{1}{2}$ in.

21. I bought $\frac{3}{8}$ yd. of cloth for \$$\frac{3}{4}$; how many *cents was that* a foot? An inch?

22. What must I give for $\frac{3}{8}$ yd. of cloth at \$$\frac{3}{4}$ a yard? How many cents will that be?

23. I gave \$$\frac{1}{11}$ for $3\frac{2}{3}$ in. of velvet; how much was that a yard? How many cents?

NOTE.—Let the answers of the following be given in dollars and cents.

24. What will $\frac{4}{7}$ of a yard of cloth cost at \$$\frac{3}{5}$ per quarter?

25. $\frac{3}{7}$ of a yard of cloth cost \$$\frac{3}{9}$? how much is that a yard?

51. LAND MEASURE—*Table.*

1 acre = 4 qr. a. = 160 sq. rd.
1 qr. a. = 40 sq. rd. = 1210 sq. yd.
1 sq. rd. = $30\frac{1}{4}$ sq. yd. = $272\frac{1}{4}$ sq. ft.
1 sq. yd. = 9 sq. ft. = 1296 sq. in.
1 sq. ft. = 144 sq. in.

26. What will $\frac{3}{7}$ of an acre be worth at \$40 per acre?

27. There is a piece of land, 20 rd. by 30; how much is it worth at \$320 per acre?

ANAL. STEPS.—1. *Find the cost of* 1 *sq. rd.*
2. *Find the cost of the land.*

28. $\frac{5}{8}$ of a rod of land cost \$$\frac{2}{7}$: what is that per acre?

52. CUBIC MEASURE—*Table.*

1 cord = 128 cu. ft.
1 cu. yd. = 27 " "
1 perch of stone = $24\frac{3}{4}$ " "
1 cu. ft. = 1728 cu. in.

Note.—A pile of wood, 8 ft. long, 4 ft. high and 4 ft. wide is a cord of wood.

29. What will $\frac{3}{8}$ of a cord of wood cost at \$5 per cord?

30. How much will 16 cu. ft. of wood cost at \$6 per cord?

Anal. Steps.—1. *Find cost of* 1 *cu. ft.*
2. *Find cost of* 16 *cu. ft.*

How many cords in a pile of wood 18 ft. long 4 ft. wide and 6 ft. high?

Form.—1. *Since a cord is* 8 *ft. in length, a pile* 18 *ft. long will be* $\frac{18}{8}$, *or* $\frac{9}{4}$ *of the length of a cord.*

2. *Since a cord is* 4 *ft. in height, a pile* 6 *ft. high will be* $\frac{6}{4}$, *or* $\frac{3}{2}$ *of the height of a cord.*

3. *Since a cord is* 4 *ft. in width, a pile* 4 *ft. wide will be once the width of a cord.*

4. *Since the pile is* $\frac{9}{4}$ *of the length of a cord,* $\frac{3}{2}$ *of the heighth of a cord, and once the width of a cord, the contents will be the product of the length, heighth and width; which is* $\frac{9}{4} \times \frac{3}{2} \times 1 = \frac{27}{8} = 3\frac{3}{8}$ *cord.*

(*a.*) **Rule.**—*Divide the length by* 8, *the height by* 4, *and the width by* 4; *and the product of these fractions will be the contents of the pile in cords.*

31. How much wood in a pile 12 ft. long 6 ft. wide and 18 ft. high? Ans.—$\frac{3}{2} \times \frac{3}{2} \times \frac{9}{2} = \frac{81}{8} = 10$ cds.

32. How much must I give for a load of woo 6 ft. long 8 ft. wide and 4 ft. high at \$5 per cord?

33. What is the cost of a load of wood 9 ft. lon *5 ft. wide, 6 ft.* high, at \$8 per cord?

53. LIQUID MEASURE—*Table.*

1 gallon = 4 qt. = 8 pt. = 32 gi.
1 qt. = 2 pt. = 8 "
1 pt. = 4 "

34. What must I give for $3\frac{1}{2}$ gallons of vinegar, at 13 cents per quart? At 5 cents per pint?

35. I gave 10 ct. for $\frac{2}{3}$ of a pint of molasses; how much was that per gallon?

36. $2\frac{1}{2}$ pints of milk are worth 5 cents, how much is that a gallon?

54. DRY MEASURE—*Table.*

1 bu. = 4 pk. = 32 qt. = 64 pt.
1 pk. = 8 " = 16 "
1 qt. = 2 "

37. How many pints in $\frac{3}{4}$ of a peck of potatoes?

38. How many quarts in $\frac{5}{7}$ of a bushel of beans?

39. How much will $\frac{3}{4}$ of a quart of chestnuts cost at $4 per bushel?

40. 1 is what part of 6? Ans.—$\frac{1}{6}$.

41. 3 is what part of 4? Ans.—$\frac{3}{4}$.

42. 16 ounces are how many times 5 ounces? Ans.—$\frac{16}{5}=3\frac{1}{5}$.

43. A pint is what part of a quart?

Form.—*Since* 2 *pints make* 1 *quart*, 1 *pint is* $\frac{1}{2}$ *of a quart.*

44. 3 quarts is what part of a bushel? Of a peck?

45. 1 foot is what part of a yard? Of a rod?

46. 2 yards is what part of a rod?

55. TIME—*Table.*

1 year $=52\frac{1}{7}$ wk. $=365$ days.
1 month $=4\frac{2}{7}$ wk. $=30$ "
1 week $=7$ "

47. 1 day is what part of a month?
48. 3 days is what part of a month?
49. 5 months is what part of a year?
50. The interest of \$100 for 1 year is \$7; what is it for $2\frac{1}{6}$ months?

LESSON LXII.

MISCELLANEOUS PROBLEMS.

1. $\frac{1}{2}$ of 8 is $\frac{1}{4}$ of what number?
2. $\frac{1}{3}$ of 6 is $\frac{1}{5}$ of what number?
3. $\frac{1}{5}$ of 10 is $\frac{1}{6}$ of what number?
4. $\frac{5}{10}$ of 20 is $\frac{1}{5}$ of what number?
5. $\frac{3}{7}$ of 28 is $\frac{4}{7}$ of what number?
6. $\frac{5}{7}$ of 14 is $\frac{2}{3}$ of what number?
7. $\frac{5}{8}$ of 1 is $\frac{5}{7}$ of what number?
8. $\frac{3}{4}$ of 3 is $\frac{2}{3}$ of what number?
9. $\frac{6}{7}$ of $\frac{1}{2}$ is $\frac{3}{5}$ of what number?
10. $\frac{3}{4}$ of $\frac{2}{3}$ is $\frac{4}{5}$ of what number?
11. $\frac{1}{6}$ of $2\frac{1}{3}$ is $\frac{7}{2}$ of what number?
12. $\frac{3}{4}$ of $4\frac{6}{7}$ is $\frac{5}{2}$ of what number?
13. A man bought a barrel of flour, for $\$6\frac{1}{4}$; finding it damaged, after using $\frac{2}{7}$ of it, he returned the remainder; what should he pay for what he used?

14. In a school, $\frac{7}{8}$ of the pupils are studying arithmetic; $\frac{3}{4}$ of those studying arithmetic are studying geography; $\frac{2}{3}$ of those studying geography are studying book-keeping; what part of the pupils are studying book-keeping?

15. A painter had $\frac{5}{6}$ of a keg of white lead, worth \$$2\frac{3}{4}$ a keg; he used $\frac{4}{5}$ of what he had; what part of the keg did he use; and what was its value?

16. The hour-hand of a clock moves over 5 minute spaces in one hour; how many will it move over in $7\frac{4}{5}$ hours?

17. Two boys, 55 miles apart, are traveling toward each other: one travels at the rate of 9 miles an hour; and the other at the rate of 6 miles an hour; how long before they will meet?

18. A little girl had \$3 in her savings bank; if she saved $\frac{1}{10}$ of a dollar each week, how many weeks had she been saving her money?

19. Two men are 15 miles apart and are traveling in the same direction: the hindmost travels 10 times as fast as the other: how far must he travel before overtaking the other?

20. If \$125 will pay for 25 weeks board for 1 person, how long will it board 12 persons?

21. If a line 20 feet in length is divided into equal parts by 10 knots, what is the length of each part?

Form.—*If there are ten points of division there must be eleven parts, each of which is* $\frac{1}{11}$ *of* 20 *feet, which is* $\frac{20}{11}=1\frac{9}{11}$ *feet.*

22. A man has ten pins, with which he is to measure off 20 ft., and divide it into equal parts. What will be the length of each interval?

Form.—*Since 2 of the pins are used to mark the distance of 20 ft. there can be but 8 points of division and 9 intervals; therefore each part will be $\frac{1}{9}$ of 20 feet, which is $2\frac{2}{9}$ ft.*

23. 12 balls were fired from a cannon in 40 minutes, at equal intervals of time; how many minutes between the discharges?

24. How many ornaments are required on the cornice on the front of a house 22 ft. long, the centre of each ornament to be 3 ft. distance from each other, and the first and last to be $\frac{1}{2}$ ft. from the corner?

25. What must be the intervals between the centres of 8 ornaments on the cornice on the front of a house 30 ft. long, if the first and last are to be placed 3 ft. 3 in. from the corner?

26. A merchant wished to place 20 stools for the use of his customers at equal intervals along his counter; the distance from the first to the last is 71 feet; how far apart are the centres?

27. A starts from a place 12 miles behind B, but A travels at the rate of 12 miles an hour, while B travels only 1 mile; in how many hours will A overtake B? How far must A travel before he overtakes B? How far will B travel before A overtakes him?

28. The hour and minute hands of a clock are exactly together at 12 o'clock, when will they be together again?

PERCENTAGE.

LESSON LXIII.

56. Per cent. signifies *by the hundred;* therefore 3 pounds gain on 100 pounds is said to be a gain of 3 per cent., or of 3 *hundredths* of a pound on 1 pound. A loss of 5 *hundredths* of a gallon is said to be a loss of 5 per cent.

The words *per cent.* are usually represented by the following character (%) thus, 3% is read 3 per cent.

1. If 3 per cent. of $1 is 3 *hundredths* of a dollar, what is 3% of $40.

FORM.—*Since* 3 *per cent. of* $1 *is* 3 *hundredths of a dollar,* 3 *per cent. of* $40 *is* 40 *times* 3 *hundredths, which is* 120 *hundredths, equal to* $1.20.

2. 46 hundredths of a dollar equal what? ANS. 46 cents.

3. 260 hundredths of a dollar equal what? ANS. $2.60.

4. $\$\frac{476}{100}$ = what?
5. $\$\frac{37}{100}$ = what?
6. $\$\frac{43}{100}$ = what?
7. $\$\frac{34}{100}$ = what?
8. $\$\frac{23}{100}$ = what?
9. $\$\frac{24}{200}$ = what?
10. $\$\frac{2}{100}$ = what?
11. $\$\frac{6}{100}$ = what?

12. $\$\frac{467}{100}$ = what?   14. $\$\frac{273}{100}$ = what?

13. $\$\frac{371}{100}$ = what?   15. $\$\frac{420}{100}$ = what?

16. What is 2% of $5? Ans.—$\$\frac{10}{100}$=10 cts.

17. What is 3% of $16? Of $11? Of $13? Of $17?

18. What is 5% of $12? Of $17? Of $200? Of $1?

19. What is 12% of $3? Of $1? Of $\$\frac{1}{2}$? Of $\$\frac{2}{3}$?

20. What is 7% of 40 pounds of sugar?

Form.—*Since 7% of 1 pound is 7 hundredths, 7% of 40 lb. is 40 times 7 hundredths, which is 280 hundredths of a pound, equal to* $2\frac{80}{100}=2\frac{4}{5}$ *lb.*

21. What is 5% of 3 ounces of indigo?

22. What is 11% of 30 yd. of cloth?

23. 25% is what part of a number? 30%? 10%? 50%? 75%? 20%? $33\frac{1}{3}$%? $12\frac{1}{2}$%?

Form.—*25% is* $\frac{25}{100}$ *equal to* $\frac{1}{4}$ *of the number.*

SLATE AND BLACKBOARD EXERCISES.

MODEL.	(1.)	(2.)	(3.)	(4.)	(5.)
30%=$\frac{3}{10}$	48%	500%	40%	604%	805%
24%=$\frac{6}{25}$	36%	660%	27%	310%	25%
32%=$\frac{8}{25}$	507%	320%	36%	407%	$33\frac{1}{3}$%
430%=$4\frac{3}{10}$	607%	2%	25%	502%	$12\frac{1}{2}$%
678%=$6\frac{39}{50}$	204%	7%	40%	606%	$16\frac{2}{3}$%
487%=$4\frac{87}{100}$	610%	60%	120%	27%	$66\frac{2}{3}$%

24. What is 8% of 28 yards of cloth?

25. What is 3% of $33\frac{1}{3}$ pounds of sugar?

26. What is 15% of 110 barrels of salt?

27. What is $\frac{2}{3}$% of $25?

FORM.—*Since* 1% *of* $25 *is* 25 *cts.*, $\frac{2}{3}$ *of* 1% *is* $\frac{2}{3}$ *of* 25 *cts.*, *which is* $\frac{50}{3}=16\frac{2}{3}$ *cts.*

28. What is 1% of $20? Of $5? Of $6?

29. What is $\frac{3}{4}$% of $20? Of $5? Of $30?

30. What is $\frac{3}{5}$% of 30 barrels of oil?

31. What is $\frac{1}{8}$% of 70 cwt. of coal?

32. What is $\frac{3}{4}$% of 1000 bushels of wheat?

33. What is 1% of $\$\frac{1}{2}$? Of $\$\frac{1}{4}$? Of $\$\frac{3}{4}$? Of $\$\frac{2}{5}$?

34. What is $\frac{3}{4}$% of $\$\frac{1}{2}$? Of $\$\frac{1}{4}$? Of $\$\frac{3}{4}$?

35. A man bought 75 barrels of apples, and opening them found 8% of them rotten; how much did he lose?

36. I had $400 in a bank; drew out 12% of it: how much remains?

37. How many *hundredths* in $1? ANS.—100 *hundredths.*

38. How many *hundredths* in $\$2\frac{1}{2}$? ANS.—100 *hundredths.*

39. What is 100% of a number? ANS.—100 *hundredths, or the whole of it.*

40. What is 200% of a number? ANS.—$\frac{200}{100}$, *or* 2 *times the number.*

41. What is 240% of a number? ANS.—$\frac{240}{100}$, *or* $2\frac{2}{5}$ *times the number.*

42. A man had 30% of his money stolen; how much had he left?

FORM.—*If he had* 100%, *and had* 30% *stolen, he must have had left the difference between* 100% *and* 30% *which is* 70%.

43. A boy had 14% of a certain sum given him

at one time, and 13% of it at another time; what per cent. of it did he receive?

44. What is the sum of 3%, 5%, 7%, and 11% of a certain sum of money? What per cent. remains?

45. I gave away 25% of my salary, and spent 11% of it; what per cent. remained?

46. What is the difference between 30% and 25% of a quantity of flour?

47. 20% is what part of a quantity?

FORM.—*20% is $\frac{20}{100}$ of the quantity, and $\frac{20}{100}=\frac{1}{5}$.*

48. 30% is what part of a quantity? 25%? 30%? 75%? 80%? 200%? 250%? 50%?

49. $\frac{2}{3}$ of a quantity is what per cent of it.?

FORM.—*Since* 100 *per cent. is the whole of the quantity, $\frac{2}{3}$ of the quantity will be $\frac{2}{3}$ of* 100%, *which is $\frac{200}{3}=66\frac{2}{3}$.*

50. $\frac{3}{4}$ of a quantity is what per cent.? ANS.—75%.

51. $\frac{4}{5}$ of a quantity is what per cent. of it?

52. $\frac{3}{5}$ of a barrel of flour is what per cent. of it?

53. $\frac{3}{4}$ is what per cent.? $\frac{5}{8}$? $\frac{6}{7}$? $\frac{2}{3}$? $\frac{4}{7}$? $\frac{3}{5}$?

54. $\frac{3}{10}$ is what per cent.? $\frac{7}{10}$? $\frac{1}{10}$? $\frac{9}{10}$? $\frac{1}{3}$?

55. $\frac{1}{4}$ is what per cent.? $\frac{2}{5}$? $\frac{4}{5}$? $\frac{5}{6}$? $\frac{1}{6}$? $\frac{1}{3}$? $\frac{3}{8}$?

56. $\frac{7}{8}$ is what per cent.? $\frac{1}{20}$? $\frac{1}{15}$? $\frac{1}{18}$? $\frac{3}{20}$? $\frac{2}{25}$?

SLATE AND BLACKBOARD EXERCISES.

MODEL.	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
$\frac{3}{4}=75\%$	$\frac{6}{7}=$?	$\frac{3}{20}$	$\frac{2}{11}$	$\frac{2}{3}$	$\frac{5}{13}$	$\frac{11}{12}$	$\frac{14}{50}$
$\frac{3}{20}=15\%$	$\frac{4}{7}=$?	$\frac{4}{35}$	$\frac{11}{13}$	$\frac{3}{2}$	[illegible]	$\frac{12}{5}$	$\frac{4}{25}$
$\frac{2}{7}=28\frac{4}{7}\%$	$\frac{3}{8}=$?	$\frac{6}{25}$	$\frac{4}{7}$	$\frac{6}{13}$	$\frac{7}{2}$	$\frac{5}{12}$	$\frac{11}{75}$
$\frac{3}{25}=32\%$	$\frac{4}{11}=$?	$\frac{25}{34}$	$\frac{7}{4}$	$\frac{2}{7}$	$\frac{1}{6}$	$\frac{6}{7}$	$\frac{7}{12}$
$\frac{7}{30}=23\frac{1}{2}\%$	$\frac{2}{3}=$?	$\frac{6}{17}$	$\frac{16}{9}$	$\frac{4}{9}$	$\frac{6}{7}$	$\frac{6}{5}$	$\frac{8}{9}$
$\frac{7}{2}=350\%$	$\frac{5}{7}=$?	$\frac{5}{13}$	$\frac{9}{16}$	$\frac{2}{9}$	$\frac{3}{4}$	$\frac{13}{15}$	$\frac{3}{13}$

57. What part of 28 gallons is 30% of it? Ans.—$\frac{3}{10}$ *of* 28 *gal.*

58. What part of 60 sheep is 75% of them? Ans.—¾ *of* 60 *sheep.*

59. What part of 3 lb. is 10% of it? 12%? 40%?

60. What part of $25 is 11% of it? 32%? 15%?

61. What part of a number is 12½% of it? 6¼%? 16⅔%? 20%? 37½%? 33⅓%?

LESSON LXIV.

1. A boy had 60 marbles, he lost 40% of them; what part of his marbles did he lose, and how many had he left?

2. I sold ⅔ of a barrel of flour; what per cent. of it did I have left?

3. I bought some cheese: ⅚ of it was good; the rest was worthless; what per cent. did I lose?

4. A man had 20 barrels of potatoes; 4 bbl. were rotten; what per cent. did he lose?

Form.—*If he had* 20 *barrels, and lost* 4 *barrels, he must have lost* $\frac{4}{20}$, *or* ⅕, *of his potatoes; and* ⅕ *of* 100% *is* 20 *per cent.*

5. 5 is what per cent. of 20? Of 40? Of 15?

Form.—5 *is* $\frac{5}{20}$ *or* ¼ *of* 20; ¼ *of a quantity is* 25% *of it.*

6. $20 is what per cent. of $30? Of $40? Of $100?

7. 6 lb. is what per cent. of 20 lb.? Of 30 lb.? Of 95 lb.?

8. What per cent. of 24 ounces are 3 oz.? 12 oz.? 6 oz.?

9. What per cent. of 8 miles are 3 miles? 4 miles? 6 miles?

10. What per cent. of 4 is 1? 3? 4? 6? 8?

11. What per cent. of 14 cents are 6 cents? 2 cts.? 1 ct.?

12. What per cent. of \$6 is \$$\frac{2}{3}$? \$$\frac{3}{4}$? \$$\frac{7}{8}$? \$$\frac{5}{9}$?

FORM.—*Since* \$1 *is* $\frac{1}{6}$ *of* \$6, \$$\frac{2}{3}$ *is* $\frac{2}{3}$ *of* $\frac{1}{6}$ *or* $\frac{1}{9}$; $\frac{1}{9}$ *of a quantity is* 11$\frac{1}{9}$%; *therefore* \$$\frac{2}{3}$ *is* 11$\frac{1}{9}$% *of* \$6.

13. What per cent. of 5 cents are 3 cts.? 1 ct.? $\frac{3}{4}$ ct.?

14. What per cent. of \$5 is \$1? \$$\frac{2}{3}$? \$2$\frac{3}{4}$? \$1$\frac{3}{5}$?

15. What per cent. of \$1 is \$$\frac{3}{4}$? \$$\frac{2}{3}$? \$$\frac{2}{5}$? \$$\frac{7}{8}$?

FORM.—*Since* \$1 *is* 100% *of* \$1, $\frac{3}{4}$ *is* $\frac{3}{4}$ *of* 100%, *which is* 75%, *therefore* \$$\frac{3}{4}$ *is* 75% *of* \$1.

16. What per cent. of \$$\frac{3}{4}$ is \$$\frac{2}{5}$? \$$\frac{3}{5}$? \$$\frac{4}{6}$? \$$\frac{6}{7}$?

FORM.—\$$\frac{3}{5}=\frac{8}{20}$; \$$\frac{3}{4}=\frac{15}{20}$; \$$\frac{8}{20}$ *is* $\frac{8}{15}$ *of* \$$\frac{15}{20}$; *and* $\frac{3}{15}$ *of a quantity is* 53$\frac{1}{3}$% *of it; therefore* \$$\frac{2}{5}$ *is* 53$\frac{1}{3}$% *of* \$$\frac{3}{4}$.

LESSON LXV.

1. 3 quarts are what per cent. of 3 gallons? Of 2 gal.? Of 6 gal.?

FORM.—3 *gallons*=12 *qt.* 3 *qt. is* $\frac{3}{12}$, *or* $\frac{1}{4}$, *of* 12 *qt.;* $\frac{1}{4}$ *of a quantity is* 25%; *hence* 3 *qt. are* 25% *of* 3 *gal.*

NOTE.—Reduce denominate numbers to the same denomination, and fractions to a common denominator.

2. From a hogshead of molasses, a grocer drew 9 gallons; what per cent. did he draw; and what per cent. remained?

3. From a box of tea containing 80 pounds, there were sold at one time 30 lb.; at another, 12 lb.; what per cent. was sold?

4. From a heap of potatoes 2 bu. of every 3 were bad; what per cent. was bad? What per cent. was good?

5. A man received $4 per year for the use of $50; what per cent. did he receive for its use?

6. I had $500 in a bank; I drew out 12½%; how many dollars have I still in the bank?

7. I bought a horse for $200, and sold it for $220; how much did I make? What per cent. of the cost did I make?

8. A grocer having a barrel of sugar weighing 240 lb., sold ¼ of it at one time, and ⅛ of it at another time; what per cent. remained unsold?

9. I receive $5 for collecting a note for $75; what per cent. of the note do I get? What per cent. does the owner of the note receive?

10. A man gave me 2% for collecting $550; how much did I receive?

FORM.—*If I am to receive* 2 *hundredths of a dollar for collecting* $1, *for collecting* $550 *I shall receive* 550 *times* 2 *hundredths of a dollar, which is* $11.

11. What must be paid for collecting a tax of $550 at a commission of 5%?

12. An agent sells $1200 worth of produce, what does his commission amount to at 5%?

FORM.—*5% equals* $\frac{1}{20}$ *of the quantity;* $\frac{1}{20}$ *of* $1200 *equals* $60; *therefore his commission amounts to* $60.

13. My agent sold $15000 worth of goods. I pay him 12½%; to how much is he entitled?

14. I paid a broker ⅛% for buying $64 worth of gold; to how much did his commission amount?

15. A man is able to pay only 40% of his debts; he owes me $500; how much is my debt worth?

16. A broker charged me ¾% for selling $600 of Albany and Susquehanna R. R. stock; what was the amount of his bill?

17. A real estate agent charges 1½% for selling my house for $5000; how much should he retain? How much should he pay me?

18. At ⅘%, what will be the expense of buying a bill of exchange of $625?

19. A boy bought a book for 30 cents and sold it for 10 per cent. advance; what did he receive for it?

FORM.—10% *is* $\frac{1}{10}$ *of the cost;* $\frac{1}{10}$ *of* 30 *cts. is* 3 *cts., which is the amount of gain;* 30 *cts.* + 3 *cts.* = 33 *cts.; hence he received* 33 *cts.*

20. A merchant bought a piece of cloth for $80, and sold it at a gain of 5%; how much did he receive?

21. I gained 20% on butter that cost $200; what did I sell it for?

22. I bought 20 lb. of cheese at 20 cts. per pound, and sold it at an advance of 5%; what did I get for it?

23. I bought a cow for $40 and sold her for $50; what did I make per cent.?

Anal. Steps:—1 *Find the amount gained.*
2. *Find the amount per cent.*

Form.—If I gave $40 and sold her for $50, I gained $10; and $10 is $\frac{10}{40}$, or $\frac{1}{4}$, of the cost; $\frac{1}{4}$ equals 25%: therefore I gained 25%.

(*a.*) **Note.**—Gain or loss is always estimated on the cost of the article bought or sold.

The *cost* plus the *gain* equals *selling price.*

The *selling price* less the *gain* equals the *cost.*

The *selling price* less the *cost* equals the *gain.*

24. If I buy milk for 8 cts. per quart, and sell it for 10 cts., what per cent. do I make?

25. A farmer bought a yoke of oxen for $75, he paid $25 for keeping them, and then sold them for $150, what per cent. did he make?

26. A grocer bought molasses at 75 cts., and sold it for 80 cts., what per cent. did he make?

27. Bought muslin for 30 cts., and sold it for 20 cts., did I make or lose, and what per cent.?

28. A boy bought a pencil for 4 cts., and sold it for twice as much as he gave for it; what per cent. did he make?

29. A boy bought a pencil for 4 cts., and sold it for one-half of what he gave for it; what per cent. did he lose?

LESSON LXVI.

1. A boy bought a sled for $4; he was offered 12% for his bargain; how much would he have received?

FORM.—*Since he was offered* 12% *of* $4, *or* 48 *cts., he could have taken the sum of* $4 *and* 48 *cts., which is* $4.48.

2. I have sugar for which I gave $20; for how much must I sell it to gain 30%?

3. I have sugar for which I gave 10 cts. per pound; for how much must I sell it to gain 10%?

4. A speculator bought a house and lot for $8000; he sold it at 12% advance; what did he receive for it?

5. I have goods marked at an advance of 25 %; they cost 40 cts. per yd.; what are they marked per yard?

6. I have goods which cost 12 cts. per yd.; I wish to gain 30%; what must I sell them for?

7. I have goods which cost 12 cts. per yard; if I am willing to lose 25%, what must I sell them for?

8. I have steel pens which are marked $3 per gross; I wish to make a discount of 12%; what is the price?

9. I sold cravats for $4, at a loss of 20%; what was the cost?

FORM.—1. *To lose* 20% *I must have sold them for the difference between* 20% *and* 100%, *which is* 80%.

2. *If* $4 *is* 80 *per ct. of the cost,* 1% *is* $\frac{1}{80}$ *of* $4, *or* 5 *cts.;*

and 100%, or the cost, must be 100 times 5 cts., which is $5; therefore they cost $5.

10. I sold apples for $8 per barrel, at a loss of 10%; what did they cost me?

11. Sold a horse for $120, at a loss of 10%; what did it cost?

12. I sold 600 pounds of tin for $300, at a discount of 20% from the net price; what was the net or cash price?

13. I sold lace at 80 cts. per yard, at a gain of 50%; what did it cost me?

Form.—1. *To gain 50% I must have sold it for the sum of 100% and 50% of the cost which was 150% of the cost.*

2. *If 80 cts. is 150% of the cost 1% is* $\frac{1}{150}$ *of 80 cts., or* $\frac{80}{150}=\frac{8}{15}$, *and 100% of the cost is 100 times* $\frac{8}{15}=53\frac{1}{3}$ *cts.; hence the lace cost* $53\frac{1}{3}$ *cts.*

14. Sold molasses for 70 cts., which was 20% more than it cost; what did I give for it?

15. I have sugar which I sell for 15 cts. per pound, which is 25% advance on cost; what did I give for it?

16. A house was sold for $8800, which was 10% in advance of cost; how much was made by the operation?

17. I sold 20 yd. of carpeting, at $3 a yard, which was 20% advance on cost; how much did I gain?

18. I sold 200 barrels of flour at $10 per barrel, which was 10% less than cost; how much did I lose by the operation?

19. Sold butter at 40 cts. per pound, which was 10% less than cost; how much did I lose on 5000 pounds?

20. My goods are marked uniformly at a *net* price of 40% advance on cost; what per cent. do I make on the cost of cloth which is marked 42 cts. *net*, if I sell it at a discount of 10% *net?*

ANAL. STEPS.—1. *Find cost of cloth.*
2. *Find the price for which it is sold.*
3. *Find the amount of gain.*
4. *Find what per cent. the gain is of the cost.*

FORM.—1. Since 42 cts. is 140 % of the cost, the cost must be 100 times $\frac{42}{140}$ cts. or 30 cts.

2. Since the cloth was sold at 10% discount, *net*, it must have been sold for 42 cts.—4.2 cts.=37.8 cts.

3. Since the cost was 30 cts., and it was sold for 37.8 cts., the gain was 37.8—30 cts.=7.8 cts.

4. 7.8 cts. is $\frac{7.8}{30}$ of 30 cts.$=\frac{78}{300}=\frac{13}{50}=$26% of cost.

NOTE.—The following formula is a deduction from the preceding:—

FORM.—1. Since the net price is 40% advance on cost, hence it is 140% of cost.

2. If the goods were sold for 10% discount they must have been sold for 10% of 140%, which is 14%, and 140% less 14% is 126% of cost, or a gain of 26% advance on cost.

21. What is the gain per cent. on the cost of the above cloth when sold at a discount of 5% net? 10% net? 20% net?

FORM.—Since the net price is 140% *of the cost,* 5% *of the net price is* 5% *of* 140%, *which is* 7% of cost, and 140%—7%=133%

of cost, or a gain of 33%; *hence the cloth, if sold at a discount of* 5% *net will be sold at a gain of* 33% *on cost.*

LESSON LXVII.

1. How much cotton can my agent buy for $200 after deducting his commission of 10%.

FORM.—1. *If his commission is* 10% *of the money he expends, and he expends* 100% *he must receive the sum of* 100% *and* 10%, *which is* 110%.

2. *If* $200 *is* 110%, 1% *is* $\frac{200}{110}$ *or* \$$\frac{20}{11}$, *and* 100% *is* 100 *times* \$$\frac{20}{11}$ *or* \$$\frac{2000}{11}$=$181.81.

2. I sent my agent $312 with which to buy corn, after deducting his commission of 4%, what sum can he expend?

3. I gave a broker $918 to invest in U. S. Bonds; after deducting his commission of 2%, how much did he invest?

4. How many bushels of corn at $1 a bushel can an agent buy for $816, and keep 2% commission?

5. How much must be collected on a rate bill that a commission of 5% may be retained and $190 be paid over. What is the commission?

FORM.—1. *If the commission is* 5%, *and the amount collected is* 100%, *the amount paid over will equal the difference of* 100% *and* 5%, *which is* 95%.

2. *If* $190 *is* 95%, 1% *is* $\frac{1}{95}$ *of* $190, *which is* \$$\frac{190}{95}$=$2; *and* 100% *is* 100 *times* $2, *or* $200; *therefore* $200 *is to be collected, and the commission is* $10.

6. In a certain town it is necessary to raise \$3600. After deducting the collector's fees, of 10%, what amount must be assessed and collected? What is the collector's commission?

7. What premium * at 2% must be paid for insuring a house and furniture for \$8000?

FORM.—*Since the premium equals* 2% *of the sum insured, hence it equals* 2% *of* \$8000, *which is* \$160.

8. At 4%, what amount of premium must be paid to insure flour to the amount of \$1200?

9. At 2½%, what must be paid for insuring goods to the amount of \$4000?

10. A merchant values his store and goods at \$10,000; what will be the annual premium on $\frac{3}{4}$ of this value at 2½%?

11. At 2%, what amount of insurance can I obtain for \$12 annual premium?

FORM.—*Since* \$12, *the premium, is* 2%, *or* $\frac{1}{50}$, *of the amount insured, therefore the required insurance will be* 50 *times* \$12, *which is* \$600.

12. How much insurance at 1% can be obtained on a cargo of flour for \$25 premium?

13. At 10%, how much insurance can be obtained for \$50?

MISCELLANEOUS PROBLEMS.

14. A sleigh was bought for \$50, and sold for 12% *more than* it cost; what was received for it?

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* The premium is the sum paid for insurance.

15. A grocer bought some tea for $40, but finding it damaged he sold it at a loss of 12%; what did he get for it? How much did he lose? At what per cent. of the cost did he sell it?

16. Bought 6 barrels of flour for $60; for how much per barrel must it be sold to gain 10% on the cost?

17. If I buy eggs at 10 cts. per dozen, for how much must I sell them to gain 100%?

18. If I buy eggs at 10 cts. per dozen, for how much must I sell them to lose 100%?

19. If I sell eggs at 10 cts. per dozen and thereby gain 100%, what did they cost me?

20. If I can sell eggs at 10 cts. per dozen and thereby lose 100%, what did they cost me?

21. If I buy eggs at the rate of 4 for 5 cts., and sell them at the rate of 5 for 4 cts., shall I gain or lose, and what per cent.?

22. If I buy eggs at the rate of 5 for 4 cts., and sell them at the rate of 4 for 5 cts., shall I gain or lose, and what per cent.?

23. The *net* price of my goods is 90% advance on cost, what per cent. advance on cost is a discount of 10% net? 12½%? 16⅔%? 20%? 25%?

24. What per cent. on cost do I make if I sell goods at a discount of 10% and 5%* for cash? At a discount of 10%, 7½%, and 5% for cash?

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* By a discount of 10% and 5% is meant 5% discount of the remainder after 10% has been deducted. A discount of 10%, 7½% and 5% is calculated in the same manner.

LESSON LXVIII.

INTEREST.

57. **Interest** is the amount paid for the use of money.

58. The **Principal** is the sum for which interest is paid.

59. The **Rate** is the per cent. paid annually.

60. The **Amount** is the sum of the principal and interest.

(*a.*) The *interest* for one year is generally expressed at a certain number of *hundredths* of the principal. Thus the interest of any sum for one year at 1% is $\frac{1}{100}$ of the principal; and at 6% it is $\frac{6}{100}$, at 8% it is $\frac{8}{100}$ of the principal.

(*b.*) *Interest* is usually calculated at 12 months a year, and 30 days a month.

1. What is the interest of $90 for one year at 7%?

FORM.—*At 7% the interest is $\frac{7}{100}$ of the principal, $\frac{7}{100}$ of* $90 *is* $6.30.

SLATE AND BLACKBOARD EXERCISES.

2. What is the annual interest of $64 at 6%?

OPERATION.—6%=$\frac{6}{100}$; $\frac{6}{100}$ of $64=$3.84.

MODEL.	(1).	(2).
$34 at 5%=$1.70	$430 at 4%	$72 at 3½%
$20 at 3%=$0.60	$2.90 at 6%	$300 at 7%
$2.80 at 2%=$0.06*	$42 at 5%	$63 at 8½%
$.90 at 8%=87 cts.	$67 at 3%	$40 at 4¼%
$46.20 at 7%=$3.23	$20 at 5½%	$2.30 at 6%

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* When the remainder is one half-cent or more, it is customary to call it one cent.

3. What is the annual interest of $500 at 7%?

4. What is the annual interest of $5000 at 4¼%?

5. What is the monthly interest of any principal at 1%?

Form.—*Since the interest for one year at 1% is $\frac{1}{100}$ of the principal, for one month it must be $\frac{1}{12}$ of $\frac{1}{100}$, or $\frac{5}{6}$ of $\frac{1}{1000}$ of the principal.*

6. What is the monthly interest of $24.60 at 1%?

Form.—*The monthly interest is $\frac{5}{6}$ of $\frac{1}{1000}$ of the principal, $\frac{1}{1000}$ of $24.60 is 24 mills, and $\frac{5}{6}$ of 24 mills is 2 cts.*

Write the monthly interest of the following principals at 1%:

7. $36.40
8. $28.30
9. $140
10. $6.84
11. $46.20
12. $30.24
13. $896
14. $24.30
15. $671.80
16. $48.30
17. $546
18. $720
19. $960
20. $2400
21. $864
22. $9600

23. What is the monthly interest of $108.60 at 7%?

Form.—*Since the interest of* $108.60 *at* 1% *is* 90 *m. or* 9 *cts., the interest at* 7% *is* 7 *times* 9 *cts., which is* $.63

24. What is the monthly interest of $144.60 at 5%? At 3%? At 2%?

25. What is the monthly interest of $960.24 at 8%? At 5½%? At 3⅛%? At 2¼%?

26. What is the monthly interest of $860 at 7%? At 2¼%? At 2⅜%?

27. What is the daily interest of any principal at 1%.

Form.*—*Since the monthly interest is $\frac{5}{6}$ of $\frac{1}{1000}$ part of any principal, the daily interest will be $\frac{1}{30}$ of $\frac{5}{6}$, equal to $\frac{1}{36}$ of $\frac{1}{1000}$ of the principal.*

28. What is the daily interest of $3600 at 1%?

Form.—*The daily interest is $\frac{1}{36}$ of $\frac{1}{1000}$ part of the principal; $\frac{1}{1000}$ of* $3600 *is* $3.60, *and $\frac{1}{36}$ of* $3.60 *is* 10 *cts., therefore the daily interest of* $3600 *at* 1% *is* 10 *cts.*

SLATE AND BLACKBOARD EXERCISES.

Write the daily interest of each of the following principals:

29. $3806
30. $4187
31. $7234
32. $8486
33. $1137
34. $726.73
35. $41378
36. $6437
37. $4137
38. $6271
39. $4137
40. $6834
41. $7342
42. $6732
43. $41672
44. $560
45. $890
46. $678
47. $417.60
48. $603.20

49. What is the daily interest of $864 at 7%?

Operation.—$\frac{1}{36}$ of $\frac{1}{1000}$ of 864=.024\times 7=17$ cts.

Form.—*The daily interest at* 1% *is* 24 *mills, and at* 7% *it is* 7 *times* 24 *mills*=17 *cts.*

50. What is the daily interest of $86 at 8%? At 5%? At 4%?

51. What is the daily interest of $743.30 at 6½%? At 8¼? At 7%? At ¾%?

52. What is the daily interest of $43.60 at 8½%? At 3⅛%? At 6¼%?

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* **Note.**—This formula gives the daily interest at 360 days to the year; but to find the interest at 365 days, subtract $\frac{1}{73}$ of the interest from itself. The difference on small sums is practically of no value, but on large principals it is a matter of considerable importance. In New York the interest is usually calculated at 365 days, although 360 is the legal rate (Revis. Stat., vol. ii., p. 182).

LESSON LXIX.

61. (*a.*) Since on any principal,
At 1% the annual interest $=\frac{1}{100}$ of the principal;
At 1% the monthly interest $=\frac{5}{6}$ of $\frac{1}{1000}$ of the principal;
At 1% the daily interest $=\frac{1}{36}$ of $\frac{1}{1000}$ of the principal;

(*b.*) Hence:
At 6% the annual interest $=6$ times $\frac{1}{100}$ of the principal;
At 6% the monthly interest $=6$ times $\frac{5}{6}$ of $\frac{1}{1000}$, or 5 times $\frac{1}{1000}$ of the principal;
At 6% the daily interest $=6$ times $\frac{1}{36}$ of $\frac{1}{1000}=\frac{1}{6}$ of $\frac{1}{1000}$ of the principal;

(*c.*) Hence:
At 7% the annual interest is 7 times $\frac{1}{100}$ of the principal;
At 7% the monthly interest is 7 times $\frac{5}{6}$ of $\frac{1}{1000}=5\frac{5}{6}$ times $\frac{1}{1000}$ of the principal;
At 7% the daily interest is 7 times $\frac{1}{36}$ of $\frac{1}{1000}=\frac{7}{36}$ of $\frac{1}{1000}$ of the principal;

(*d.*) Hence:
For 2 years at 6%, the interest $=2$ times 6 times $\frac{1}{100}$ of the principal;
For 2 months at 6%, the interest $=2$ times 5 times $\frac{1}{1000}$ of the principal;
For 2 days at 6%, the interest $=2$ times $\frac{1}{6}$ of $\frac{1}{1000}$ of the principal;

(*e.*) Hence:
For 3 years at 7% the interest is 3 times 7 times $\frac{1}{100}$ of the principal;
For 3 months at 7% the interest is 3 times $5\frac{5}{6}$ times $\frac{1}{1000}$ of the principal;
For 3 days the interest is 3 times $\frac{7}{36}$ of $\frac{1}{1000}$ of the principal.

1. What is the interest of $30.80 for 3 yr. at 6%? For 5 yr.? For 6 yr.? For 2 yr.?

MODEL OPERATION.

The interest for 3 yr. at 6%=3 times 6 times 308 mills=$5.54.

FORM.—*Since the interest of* $30.80 *for* 1 *year at* 1% *is* 308 *mills* (see 161, *a.*) *the interest for* 3 *years at* 6% *will be* 3 *times* 6 *times* 308 *mills, equal to* $5.54.

NOTE.—The teacher should require each of the following problems to be written on, and solved from the blackboard, before the mental solution is required.

2. What is the interest of $40.20 for 3 yr. at 5%? At 3%? At 7%? At 4%?

3. What is the interest of $36.50 for 2 yr. at 7%? At 3%? At 6%? At 8%?

4. At 7% what is the interest of $44.37 for 3 mo.?

MODEL OPERATION.

Int. at 7% for 3 mo.=7 times 3 times $\frac{5}{6}$ of 44 mills=77 cts.

FORM.—1. *Since the interest of* $44.37 *for* 1 *mo. at* 1% *is* $\frac{5}{6}$ *of* 44 *mills* (see 161, *a.*), *the interest at* 7% *for* 3 *mo. is* 7 *times* 3 *times* $\frac{5}{6}$ *of* 44 *mills;* 3 *times* $\frac{5}{6}$ *is* $\frac{5}{2}$, *and* $\frac{5}{2}$ *of* 44 *mills is* 11 *cts., and* 7 *times* 11 *cts. is* 77 *cts., therefore the interest is* 77 *cts.*

2.* The interest for 1 year is 7 times 443 mills, or 310 cts., hence the interest for 3 mo. or $\frac{1}{4}$ of a year is $\frac{1}{4}$ of 310 cts., or 77 cts.

5. What is the interest of $36.40 for 6 mo. at 7%? At 3%?

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* *When the* months are a convenient fractional part of a year, it is generally *better to* use *the* second formula.

6. What is the interest of $24.86 for 7 mo. at 4%? At 3%?

7. At 6% what is the interest of $26 for 4 mo.? For 7 mo.? For 5 mo.?

8. What is the interest at 4% of $80 for 8 mo.? For 5 mo.? For 2 mo.?

9. At 6% what is the interest of $380 for 12 days? For 6 days? For 9 days? For 11 days?

MODEL OPERATION.

Int. at 6% for 12 da.= 6 times 12 times $\frac{1}{36}$ of 38 cts.=76 cts.

FORM.—*Since the interest of* $380 *at* 1% *for* 1 *da. is* $\frac{1}{36}$ *of* 38 *cts.* (see 161, *a.*), *the interest at* 6% *for* 12 *days is* 6 *times* 12 *times* $\frac{1}{36}$ *of* 38 *cts.*; 12 *times* $\frac{1}{36}=\frac{1}{3}$; 6 *times* $\frac{1}{3}$ *is* 2; 2 *times* 38 *cts. is* 76 *cts., therefore the interest is* 76 *cts.*

10. What is the interest of $36.80 at 7% for 6 days? At 9%? At 3%? At 4%?

FORM.—*The interest of* $36.80 *at* 7% *for* 6 *days is* 7 *times* 6 *times* $\frac{1}{36}$ *of* 36 *mills equal to* 42 *mills or* 4 *cts.*

11. What is the interest of $46 for 11 days at 7%? At 6%? At 3%?

12. At 7% what is the interest of $860 for 5 days? For 8 days? For 12 days? For 15 days?

13. What is the interest of $670 for 13 days at 6%? At 4%? At 5%? At 7%?

14. At 7% what is the interest of $480.67 for 2 yr. 3 mo. 12 da.

ANAL. STEPS.—1. *Find interest for* $2\frac{1}{4}$ *yr.*[*]
2. *Find interest for* 12 *da.*
3. *Find total interest.*

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[*] *NOTE.—3 mo. being a convenient part of a year, reduce it to the fraction of a year.*

MODEL OPERATION.*

Int. $480.67 at 7% for 2¼ yr.=7 times 2¼ times $4.806= $75.69

Int. $480.67 at 7% for 12 da.=7 times 12 times $\frac{1}{36}$ of 48cts= $1.12

$76.81.

FORM.—1. *The interest of* $480.67 *at* 7% *for* 2¼ *yr. is* 7 *times* 2¼ *times* $4.806, *equal to* $75.69.

2. *The interest of* $480.67 *at* 7% *for* 12 *days is* 7 *times* 12 *times* $\frac{1}{36}$ *of* 48 *cts. equal to* $1.12.

3. *If the interest for* 2¼ *yr. is* $75.69, *and for* 12 *days it is* $1.12, *for* 2 *yr.* 3 *mo.* 12 *da. it will be the sum of their amounts, which is* $76.81.

15. What is the interest of $80 for 2 yr. 7 mo. 18 da. at 8%?

MODEL OPERATION.

Int. for 2 yr.=2 times 8 times 80 cts. =$12.80
Int. for 7 mo.=7 times 8 times $\frac{5}{6}$ of 8 cts.=$3.73
Int. for 18 da.=18 times 8 times $\frac{1}{36}$ of 8 cts. =32
Total interest, $16.85

NOTE.—The following problems should be written on the slate or blackboard after the model, the pupils giving the solution mentally.

16. What is the interest of $60.80 for 5 mo. and 12 da. at 8%? For 2 yr. 8 mo. at 6%?

17. What is the interest of $3.60 for 8 mo. 24 da. at 7%? At 8%? At 6½%?

18. What is the interest of $47.60 for 11 mo. 16 da. at 7%? At 6%? At 5%?

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* *To obtain* accurate results for large principals for a long time, it is *necessary to retain the tenths* of a mill; but for small principals for an ordinary time it is not necessary.

For convenience in finding the number of days between two dates we give the following

TABLE (*f*.)

Jan.	Feb.	Mar.	Apr.	May.	June	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1	32	60	91	121	152	182	213	244	274	305	335
2	33	61	92	122	153	183	214	245	275	306	336
3	34	62	93	123	154	184	215	246	276	307	337
4	35	63	94	124	155	185	216	247	277	308	338
5	36	64	95	125	156	186	217	248	278	309	339
6	37	65	96	126	157	187	218	249	279	310	340
7	38	66	97	127	158	188	219	250	280	311	341
8	39	67	98	128	159	189	220	251	281	312	342
9	40	68	99	129	160	190	221	252	282	313	343
10	41	69	100	130	161	191	222	253	283	314	344
11	42	70	101	131	162	192	223	254	284	315	345
12	43	71	102	132	163	193	224	255	285	316	346
13	44	72	103	133	164	194	225	256	286	317	347
14	45	73	104	134	165	195	226	257	287	318	348
15	46	74	105	135	166	196	227	258	288	319	349
16	47	75	106	136	167	197	228	259	289	320	350
17	48	76	107	137	168	198	229	260	290	321	351
18	49	77	108	138	169	199	230	261	291	322	352
19	50	78	109	139	170	200	231	262	292	323	353
20	51	79	110	140	171	201	232	263	293	324	354
21	52	80	111	141	172	202	233	264	294	325	355
22	53	81	112	142	173	203	234	265	295	326	356
23	54	82	113	143	174	204	235	266	296	327	357
24	55	83	114	144	175	205	236	267	297	328	358
25	56	84	115	145	176	206	237	268	298	329	359
26	57	85	116	146	177	207	238	269	299	330	360
27	58	86	117	147	178	208	239	270	300	331	361
28	59	87	118	148	179	209	240	271	301	332	362
29		88	119	149	180	210	241	272	302	333	363
30		89	120	150	181	211	242	273	303	334	364
31		90		151		212	243		304		365

NOTE.—To find from the above table the number of days between two dates, the following rule is given:

(*g*) I.—*When the dates are in the same year, subtract the number of days in the place of the earlier date from the number of days of the later date; the result will be the number of days required.*

II. When the dates are in consecutive years, subtract the number of *days in the earlier date from* 365 *and add to the remainder the number of days in the place of the later date; the result will be the number of days required.*

(*h.*) *When the year is a leap year, add one day to the result.*

How many days between Apr. 11th and Dec. 22d of the same year?

Form.—(See Table.*) Apr. 11th is the 101st day of the year, Dec. 27th is the 361st day, and the number of days between those dates is the difference of 361 and 101, which is 260 days.

19. What is the interest of $63 for 68 days at 7%?

MODEL OPERATION.

In. for 68 da. = 68 times 7 times $\frac{1}{36}$ of 63 mills. = 84 cts.

Form.—The interest at 7% for 68 days is 68 times 7 times $\frac{1}{36}$ of 63 mills; 68 times $\frac{1}{36} = 1\frac{8}{9}$ times 63 mills, which is 12 cts., and 7 times 12 cts. are 84 cts.

Note.—The above method makes 1 day $\frac{1}{360}$ part of the year; if $\frac{1}{73}$ of the interest be subtracted from itself, the remainder will be the interest at 365 days to the year.

20. What is the interest of $380.40 at 5% for 72 days?

21. What is the interest of $460 from Feb. 3 to Nov. 12, at 6%? (See Table.) 316−34=282 days.

22. What is the interest of $860.40 from Aug. 7 to Dec. 29, at 5%? From Feb. 11 to Nov. 17?

23. What is the interest of $40 from Nov. 11, 1866 to Apr. 5, 1867, at 7%? From Aug. 3, 1865 to July 1, 1866?

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[*] *The dates of* each month correspond to the dates of January, in the *left-hand column.*

24. What is the amount of $30 for 3 mo. 15 da. at 6%?

FORM.—Since the interest of $30 for 3 mo. and 15 da., is 53 cts., the amount will be the the sum of the principal and interest, which is $30.53.

25. What is the amount of $40 for 6 mo. and 12 da. at 6%?

26. What is the amount of $35 from April 12 to October 11, at 6%?

27. What is the amount due on a note of $600 on interest 3 mo. 18 da., at 7%?

28. I have a note for $25, dated April 11, 1867, at 7%; what will it amount to on the 17th of Nov. following?

29. What is the interest of $26 from May 11th to October 27th, at 7%? From Sept. 12, 1865, to Apr. 7, 1867?

30. What is the *amount* of $33.20 from June 5, 1865, to Jan. 11, 1867, at 6%? From Dec. 4, 1866, to Apr. 5, 1867?

LESSON LXX.

1. What principal in 5 years at 7% will give $34 interest?

MODEL OPERATION.

Int. at 7% in 5 yr.=$\frac{7}{20}$ of principal. $\frac{1}{7}$ of 20 times $34= 97$\frac{1}{7}$=$97.14.

Form.—1. *At 7% for 1 year the interest equals* $\frac{7}{100}$ *of the principal, and for 5 years it equals 5 times* $\frac{7}{100}$, *or* $\frac{35}{100}=\frac{7}{20}$ *of the principal.*

2. *If for the given time and rate the interest equals* $\frac{7}{20}$ *of the principal, then $34, the given interest, is* $\frac{7}{20}$ *of the required principal. If $34 is* $\frac{7}{20}$, $\frac{1}{20}$ *must be* $\frac{1}{7}$ *of $34, which is* $\$4\frac{6}{7}$; *and* $\frac{20}{20}$, *or the required principal must be 20 times* $\$4\frac{6}{7}$, *or* $\$97\frac{1}{7}$ $=\$97.14$.

2. What principal in $3\frac{1}{2}$ years at 6% will give $32 interest?

Form.—*Since for* $3\frac{1}{2}$ *years at 6% the interest equals* $3\frac{1}{2}$ *times* $\frac{6}{100}$, *or* $\frac{21}{100}$ *of the principal, $32 must be* $\frac{21}{100}$ *of the required principal; hence* $\frac{1}{21}$ *of 100 times $32, or $152.38 is the required principal.*

3. What principal in 7 years at 5% will give $21 interest?

4. What principal in 8 years at 6% will give $12 interest?

5. What principal in 7 years at 4% will give $70 interest?

6. What principal in 1 yr. 8 mo. at 9% will give $36 interest?

Form.—*Since for* $1\frac{2}{3}$ *years the interest equals* $1\frac{2}{3}$ *times* $\frac{9}{100}$ *or* $\frac{45}{300}=\frac{3}{20}$ *of the principal, $36 must be* $\frac{3}{20}$ *of the required principal, which is $240.*

7. I wish to invest money in 6% U. S. bonds so that my annual interest will amount to $800. What must be the amount? What must be the amount that my quarterly interest may be $50?

8. In what time, at 6%, will $300 gain $45?

Form.—*If in* 1 *year at* 6% $300 *will gain* $18, *to gain* $45 *it will take as many years as* 18 *is contained times in* $45, *that is*, $2\frac{1}{2}$ *times, therefore it will take* 2 *years* 6 *months.*

9. In what time will $120 gain $40 at 10%?

10. In what time will $80 gain $6 at 3%?

11. In what time will any principal double itself at 4%?

Form.—*Since to double itself at* 1% *any sum of money will require* 100 *years; hence to double itself at* 4% *it will require* $\frac{1}{4}$ *of* 100 *years, which is* 25 *years.*

12. In what time will a given principal double itself at 8%? At 6%? At 9%?

13. In what time will $300 double itself at 10%? At 6%? At $6\frac{1}{2}$%?

14. In what time will $33.68 double itself at 20%? At 5%? At $12\frac{1}{2}$%?

15. A man received $24 for the use of $480 for 2 years; what was the rate per cent.?

Form.—*Since* $24, *the interest for* 2 *years, is* $\frac{24}{480}$, *or* $\frac{1}{20}$ *of the principal, for* 1 *year the interest will be* $\frac{1}{2}$ *of* $\frac{1}{20}$, *or* $\frac{1}{40}$, *or* $2\frac{1}{2}$% *of the principal.*

16. At what per cent. in $3\frac{1}{2}$ years will the interest of $200 amount to $49?

17. At what per cent. in $4\frac{1}{6}$ years will the interest of $60 amount to $25?

18. At what per cent. will $12 gain $5 in 8 years?

19. At what per cent. will a given principal *double itself* in 8 years?

FORM.—*Since the rate of interest on a given principal must be* 100% *to double in* 1 *year, to double in* 8 *years,* $\frac{1}{8}$ *of* 100%, *or* 12½% *will be required.*

20. At what per cent. will a given principal double itself in 5 years?

21. At what per cent. will $350 double itself in 6 years? In 2 yr.? In 11 yr.?

LESSON LXXI.

62. **Discount** is a deduction made from a debt not bearing interest when paid before maturity. It equals the difference between the present worth and the amount of the debt when due.

63. The **Present worth** of a note not bearing interest, is such a sum that, if put at interest, it will have produced the amount of the note by the time it falls due.

1. What is the present worth of $89 due in 2 yr. 8 mo. at 7%? What is the discount?

FORM.—1. 2 years 8 mo. equals $2\frac{2}{3}$ yr. The interest for $2\frac{2}{3}$ years at 7% equals $\frac{56}{300}$ or $\frac{14}{75}$ of the principal; and since the interest is $\frac{14}{75}$, and the principal is $\frac{75}{75}$, both will be the sum, which is $\frac{89}{75}$; hence $89 is $\frac{89}{75}$ of the principal,* or present worth, which is $75.

2 Since the discount equals the difference between the present worth and the amount, therefore it equals the difference between $89 and $75, which is $14.

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* *The terms principal* and *present worth* are equivalent and are often applied to the same quantity.

2. What is the present worth of a note of $135 due in 5 years without interest, if money is worth 7%?

3. The amount of a certain note which has been on interest 4 years and 2 months at 8% is $124.40; what is the principal?

4. What is the discount at 6% on a note of $560; payable in 2 years and 6 mo. without interest?

5. What is the difference between the interest and discount of $130 due 10 months hence at 10%?

64. Bank discount is the simple interest of the principal for the specified time and three days more.

65. The Proceeds of a note is the difference between the discount and the face.

(*a*). *The face* of a note is the amount for which it is drawn.

6. What is the bank discount of $80 for 30 days at 7%? What is the proceeds?

FORM.—1. *Since the bank discount equals the interest, it equals* 7 *times* 33 *times* (30+3) $\frac{1}{36}$ *of* 8 *cts., which is* $\frac{154}{3}$=51 *cts.*

2. Since the proceeds is the difference between the face and the bank discount, the proceeds will equal the difference between $80 and 51 cts., which is $79.49.

7. What is the bank discount of $200 for 60 days at 6%? What are the proceeds?

8. What is the bank discount of $350 for 90 days at 6%? What is the proceeds?

9. I had a note of $500, which I discounted at a bank for 60 days at 6%; what were the proceeds?

10. I sold a horse for a note for $250 at 30 days

at 7%; what is the present worth of the note discounted at a bank?

11. I was offered for my farm $2500 cash or a note for $2600 at 30 days: I took the latter, and had the note discounted at a bank at 6%; did I make or lose, and how much?

MISCELLANEOUS PROBLEMS.

12. A man owning $\frac{4}{5}$ of a mill worth $3600 sold $\frac{2}{3}$ of his share; what part of the mill does he still own? What was the value of the part which he sold?

13. A man having $2\frac{1}{3}$ barrels of flour gave $\frac{1}{3}$ of it to a poor neighbor; what part of the whole has he left? How many barrels did he give away?

14. A jeweler sold a watch for $18, which was $\frac{3}{4}$ of what it cost him; what part of the cost did he lose? What did it cost him? What per cent. did he lose?

15. I sold a horse for $80 which was $\frac{2}{5}$ of twice its cost; what did it cost? What was lost by the sale?

16. A drover after selling $\frac{3}{5}$ of his sheep, has 20 left; how many had he at first?

FORM.—1. *Since a drover had* $\frac{5}{5}$ *and sold* $\frac{3}{5}$ *he must have had left* $\frac{5}{5}$ *less* $\frac{3}{5}$, *which is* $\frac{2}{5}$ *of his sheep.*

2. Since 20 equals $\frac{2}{5}$ *of the number of sheep,* $\frac{1}{5}$ *must equal* $\frac{1}{2}$ *of 20, and* $\frac{5}{5}$ *equals 5 times* $\frac{1}{2}$ *of 20 sheep, which is 50 sheep.*

17. A pole was standing $\frac{3}{4}$ of its length in the air, $\frac{2}{3}$ of the remainder in the water, and 3 ft. in the mud; what was the length of the pole? How many feet in the air?

18. A woman bought an equal number of oranges and lemons for 80 cts.; for the oranges she paid 3 cts. each, and for the lemons 2 cts. each; how many of each did she buy?

19. What is 30% of 25 gallons of vinegar?

20. $7\frac{1}{2}$ gallons of wine is what per cent. of a cask containing 25 gallons?

21. I paid $4 a yard for a piece of cloth containing 40 yd.; $\frac{3}{5}$ of it being damaged, I sold the part damaged at a loss of 20%; how much did I receive for it?

22. I lost 3 bushels out of every 9 bushels of my potatoes; what per cent. was that?

23. A man saves $\frac{3}{4}$ of his income, what per cent. does he spend?

24. A merchant invests 60% of his money in cloths; what per cent. of it has he remaining? What part of it?

25. A debtor of mine failed, and was able to pay $3 on every $12; what per cent. did I lose on a debt of $500?

26. What will be the expense of collecting a note of $8600 at 10% commission?

27. I sent 4000 lb. of butter to an agent in New-York; he sold it at 50 cts. per pound; what did his *commission amount* to at 5%?

28. I gave a broker $\frac{3}{4}$% for selling $600 of U. S. bonds at par; how much did it cost?

29. I sent $510 to an agent to buy goods, after taking his commission of 2%, how much can he spend for me?

30. What amount must be collected on a rate bill, that the collector may retain his fee of 5%, and pay over $228?

31. I own $\frac{3}{4}$ of a boat load of corn valued at $1600; it is insured for $\frac{3}{4}$ of its value at 2%; what premium must I pay?

32. At $\frac{3}{4}$% what amount of insurance on a cargo of wheat can be obtained for $72?

33. I bought a wagon for $150, and sold it for 8% more than it cost; what did I get for it?

34. I bought a quantity of sugar for $84 and sold it at an advance of 12$\frac{1}{2}$%; what did I receive for it?

35. Bought 5 barrels of flour for $60; for what must I sell it per barrel to gain 5% on the cost?

36. I bought calico for 12 cts., and sold it for 18 cts., what per ct. did I gain?

37. What per cent can be made by buying sugar for 8 cts. and selling it for 8$\frac{1}{2}$ cts.?

38. A merchant sold damaged cloth so as to lose $\frac{1}{8}$ of the cost; what was the loss per cent?

39. A man bought a horse at 20% discount from cost, and sold it at 20% advance on cost; what per cent. was gained on the price paid?

40. A man sold a watch for $75, which was *12$\frac{1}{2}$% above* cost; what was given for it?

41. A speculator sold 50 firkins of butter for $2700, which was 10% less than cost; how much would he have received for it had he sold it at an advance of 3%?

42. I bought butter for 50 cts., and sold it for 52½ cts.; what per cent. did I make?

43. What is the annual interest of $30 at 6%?

44. What is the monthly interest of $30 at 6%?

45. What is the daily interest of $30 at 6%?

46. What part of the principal is the annual interest at 7%?

47. What part of the principal is the monthly interest at 7%?

48. What part of the principal is the daily interest at 7%?

49. What part of the principal is the interest for 3 yr. at 7%? For 5 yr.? For 2½ yr.? For 3⅜ yr.?

50. What part of the principal is the interest for 3 months at 7%? For 4 mo.? For 5½ mo.? For 7⅓ mo.?

51. What part of the principal is the interest for 36 days at 8%? For 72 days? For 39 days? For 20 days?

52. What is the interest of $380.20 for 9 mo. and 12 da. at 8%? What is the amount?

53. What principal in 3 yr. will amount to $90 at 5%.?

54. In what time will $120 gain $50.40 interest at 6%? In what time will it double itself at 8%?

55. At what per cent. will $800 double itself in 5 years? At what per cent. will it gain $90?

56. What is the simple discount of a note without interest for $560 payable in 2 yr. at 6%?

57. What is the bank discount of $570, discounting for 90 days at 6%?

58. A milkman sold milk for 10 cts. per quart, and thereby made 25%. When milk cost 2 cts. more a quart, how much water must be put in it to make the same per cent. by selling at the same price?

59. A milkman sold milk for 10 cts. a quart, and thereby made 25 per cent. When milk cost 4 cts. more a quart, how much water must be put in it so that he can sell it for 13 cts., and make 30%?

60. Two men hired a pasture for $24; one pastured 5 horses, and the other 3; what per cent. of the cost should each pay?

TEST QUESTIONS FOR EXAMINATION.

33. How are fractions expressed?

34. What does the denominator show? To what does it give the name? In how many ways can it be written?

35. What does the numerator show? How may it be written? In how many ways can [illegible] be written? Illustrate on the blackboard. In the fraction [illegible] which is the numerator? Why? Which the denominator? Why?

36. What are called the terms of the fraction? Which term is the numerator? Which the denominator? What is meant by $\frac{1}{2}$? By [illegible]? What is meant by [illegible]? By [illegible]? When no number or units are expressed or implied what is understood?

37. What is reduction? What can be done to the terms of a fraction without altering it value? Why? Change the form of [illegible] without changing its value. What is such an operation called? Show by a diagram on the blackboard that multiplying both terms of a fraction by the same number does not change its value. Write ten fractions that are equal in value to ½. What kind of fraction may ½ be called? Ans.—*A primary fraction.*

38. Change the form of 5 tenths without altering its value. What effect has dividing the terms of a fraction by the same number? Why? Illustrate the principle on the blackboard.

39. When is a fraction said to be proper? Write five proper fractions on the blackboard.

40. When are fractions said to be improper? Write three improper fractions on the blackboard? Write three proper fractions?

41. What is a mixed number? How is it written? Write a mixed number on the blackboard. Write a proper fraction. Write an improper fraction. Reduce a mixed number to an improper fraction, and give the formula. Change an improper fraction to a mixed number and give the formula.

42. When are fractions said to be expressed in their lowest terms? Illustrate the formula by a diagram. Give the rule.

43. When are fractions of the same denomination? What is meant by fractions having a common denominator? Illustrate the formula by a diagram. Give the rule. What is ¼ of 4 apples? What is meant by ⅓ of 6 apples? What is meant by ⅓ of three peaches? What is ⅓ of 3 feet? Illustrate by a digram. What is the difference between ½ of 2 apples and ⅔ of 1 apple? Are they equivalent? Are they identical? Illustrate by a diagram. What is equal to ½ of a number? What is meant by ⅔ of a number? What is equal to ⅔ of 11?

44. What is addition? What is subtraction? Can unlike fractions be added? Can unlike fractions be subtracted? When are fractions said to be alike? Add ½ and ⅓. Give formula. Subtract ⅓ from ½. Give formula. Give the brief method for finding the difference between two fractions having 1 for a numerator. Give the rule for adding two fractions having like numerators. Give the rule for finding the difference between two fractions having like numerators. Give the rule for adding or subtracting two fractions having numerators and denominators unlike. Give method for subtracting a fraction from an integral number. What is to be done when the fractional minuend is less than the fractional subtrahend?

45. What is 3 times ⅛ of an apple? What is multiplication? What is 2 times ⅔? Illustrate the formula by a diagram? What is ½ of 5 tenths of an apple? What is division? What is meant by ½ of 6 sevenths? What part of one is ⅓ of 6 sevenths? What is meant by ⅔ of a thing? What is meant by ⅔ of ⅘? What part of one is ⅔ of ⅘? What is the rule for multiplying one fraction by another? What is the difference between ½ of a number or a number divided by 2? Between ½ of a number or a number multiplied by 2. What is the difference between ½ of 2 times a number or 2 times ½ of a number? What is the difference between ⅓ of 2 times a number or ⅔ of a number? What is the meaning of the word "of" in the expression, ½ of 7? What is the difference between dividing a number by 2 and by ½? Between multiplying a number by 2 and by ½? In how many ways can a fraction be multiplied? In how many ways can it be divided? In how many ways can a number be multiplied by a fraction? In how many ways can it be divided by a fraction? [For test-problems see Exercises under each subject.]

46. Recite the equivalents of 1 pound sterling money in lower denominations. Of 1 shilling. Of 1 penny. To how many farthings are 240 pence equal? To how many pence are 48 farthings equal?

47. Recite the equivalents of 1 pound Troy weight in lower denominators. Of 1 ounce. Of 1 pennyweight. Recite the equivalents of 1 lb. Apothecaries' weight. Of 1 ℥. Of 1 ʒ. Of 1 ℈. What is the difference between 1 lb. Troy and 1 lb. Apoth.? Between 1 oz. Troy and 1 ℥ Apothecaries'? Which contains a greater number of grains? Give the equivalents of pound, ounce and dram Avoirdupois.

Ans.—1 lb. = 16 oz. = 256 dr. = 7000 gr. Troy.
1 oz. = 16 " = 437½ " "
1 d. = 27⅓ " " (nearly).
1 lb. Avoir. = 1 lb. 2½ ℥ Apoth. (nearly).
1 ℥ Apoth. = 1 oz. 1½ dr. Avoir. "

Which is the greater, a pound Avoirdupois or a pound Troy? An ounce Avoirdupois or an ounce Apothecaries? A dram Avoirdupois or a dram Apothecaries'? How can pounds Avoirdupois be changed to pounds Troy? How can drams Apothecaries' weight be changed to drams Avoirdupois?

48. Recite the equivalents of the ton in lower denominations. Of the hundredweight. Of the pound. Of the ounce. What is the use of Troy weight? Of Apothecarie's weight? Of Avoirdupois weight?

49. Recite the equivalents of 1 mile in lower denominations. Of one rod. Of 1 yard. Of 1 foot. What is the use of long measure?

50. Recite the equivalents of 1 yard cloth measure in lower denomination. Of 1 quarter. Of 1 eighth. What is the use of cloth measure?

51. Recite the equivalents of 1 acre in lower denominations. Of 1 quar. acre or rood. Of 1 sq. rod. Of 1 sq. yard. Of 1 sq. feet. What is the use of land measure?

52. Give the equivalents in lower donominations of 1 cord. Of 1 cu. yard. Of 1 perch of stone. Of 1 cu. foot. Give the rule for finding the number of cords in a pile of wood. What is the use of cubic measure?

53. Give the equivalents of 1 gallon in lower denominations. Of one quart. Of 1 pint. What is the use of liquid measure?

54. Give the equivalents of 1 bushel in lower denominations. Of 1 peck. *Of 1 quart.* What is the use of dry measure? Give the equivalents of 1 *gallon liquid* measure, and 1 gallon or 4 quarts dry measure.

Ans.—1 gal. liq. meas. = 4 qts. = 8 pts. = 32 gi. = 231 cu in.
1* gal. dry meas. = 4 qts. = 8 pts. = 32 gi.* = 268⅘ cu. in.
1 qt. dry meas. = 1 qt. 1⅓ gills liq. meas. (nearly).
6 gills* dry meas. = 7 gills " " "

55. How many days in 1 year? How many weeks in a year? How many days in a month?

56. What is meant by per cent.? What mark is used to represent the words *per cent.*?

57. What is interest? What is that called which is paid for the use of money?

58. What is the sum called for which interest is paid?

59. What is rate? What is meant by annually.

60. What is the sum of the principal and interest called? How is the interest for one year generally expressed? What is meant by 1 per cent.? 2%? 5%? 6%? 8%? How is interest usually calculated?

61. What part of the principal is the annual interest at 1%? What part is the monthly interest at 1%? What part is the daily interest at 1%? What part of the principal is the annual interest at 6%? What part is the monthly interest at 6%? What part is the daily interest at 6%? What part of the principal is the annual interest at 7%? At 9%? At 8%? What part of the principal is the monthly interest at 4%? At 8%? At 9%? What part of the principal is the daily interest at 6%? At 8%? At 4½%? At 3½%? What part of the principal is the interest at 5% for 1 year? For 2½ years? For 3 years? What part of the principal is the interest at 3% for 1 month? For 3 months? For 5½ months? For 7 months? What part of the principal is the daily interest at 7% for 8 days? For 13 days? For 11 days? For 12½ days? Give the rule for finding from the table the exact number of days between two dates.

62. What is discount? What is meant by the maturity of a note? How many days are allowed after the time of a note has expired? Ans.—*Three days.* What are these days called? Ans.—*Days of grace*

63. What is the present worth of a note not bearing interest? What is the present worth of a note bearing interest. Ans.—*The face of the note.*

64. What is bank discount? Is there any difference between true discount and bank discount?

65. What are the proceeds of a note when discounted at a bank? What is the face of a note?

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* The denominations of *gallons* and *gills* are very seldom used in dry measure, and are only used here for convenience in comparing the two *measures.*

LESSON LXXII.

MISCELLANEOUS COMPLEX PROBLEMS FOR ANALYSIS.

N. B.—The teacher will find it very much to the advantage of his pupils, to express on, and to analyze from the blackboard, many of the following problems *before* giving the solution in the usual manner.

1. What is the sum of 8, 6, 4, 3?
2. What is the sum of $\frac{3}{8}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{1}{2}$?
3. 43 is 60 less than what number?

FORM.—*If* 43 *is* 60 *less than the number, then* 43 *plus* 60 *equals the number, which is* 103.

4. 75 is 20 less than 2 times what number?

FORM.—*If* 75 *is* 20 *less than* 2 *times the number, then* $75+20=$ 2 *times the number, and the number is* $\frac{1}{2}$ *of* 95, *which is* $47\frac{1}{2}$.

5. 60 is 10 less than $\frac{1}{2}$ of what number?

ANAL. STEPS.*—1. *Find one-half the number.*
2. *Find the number.*

6. 70 is 30 less than $\frac{2}{3}$ of what number?
7. 25 is 20 less than $\frac{3}{2}$ of what number?

ANAL. STEPS.—1. *Find* $\frac{3}{2}$ *of the number.*
2. *Find* $\frac{1}{2}$ *of the number.*
3. *Find* $\frac{2}{2}$ *of the number.*

8. Henry had 25 marbles, which were 20 less than $\frac{3}{5}$ of the number that Robert had; how many had Robert?
9. What number is 33 larger than 50?
10. What number is 25 larger than 3 times 50?
11. What number is 30 times larger than $\frac{1}{5}$ of 50?

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* Require the pupil to give the right formula.

12. What number is 25 larger than $\frac{4}{7}$ of 50?

ANAL. STEPS.—1. *Find* $\frac{4}{7}$ *of* 50.
2. *Find the number.*

13. 25 is $\frac{1}{2}$ the difference between 50 and what larger number?

14. 30 is $\frac{2}{3}$ the difference between $\frac{1}{2}$ of 30 and what larger number?

15. A boy lost 30 cts., which was $\frac{5}{8}$ of the difference between $\frac{2}{3}$ of 60 and the number of cents he had left; how much money had he left?

16. From what number must 30 be taken to get 40?

17. From $\frac{1}{2}$ of what number must $\frac{1}{4}$ of 30 be taken to get 80?

18. From $\frac{5}{8}$ of what number must $\frac{5}{6}$ of 30 be taken to get 20?

ANAL. STEPS.—1. *Find* $\frac{5}{6}$ *of* 30.
2. *Find* $\frac{5}{8}$ *of the number.*
3. *Find the number.*

19. From $\frac{5}{7}$ of a flock of sheep $\frac{3}{4}$ of 40 were sold, then there were 25 left; how many sheep in the flock?

20. $\frac{3}{4}$ of 30 taken from $\frac{2}{3}$ of 50 leaves what number?

21. $\frac{1}{2}$ the difference between $\frac{3}{4}$ of 36 and $\frac{2}{9}$ of 18 equals what number?

22. $\frac{3}{5}$ of 15 is 6 more than $\frac{2}{3}$ of what number?

ANAL. STEPS.—1. *Find* $\frac{3}{5}$ *of* 15.
2. *Find* $\frac{2}{3}$ *of the number.*
3. *Find the number.*

23. A man was asked the number of dollars he had in the savings bank; he answered, "$\frac{3}{5}$ the number of cents in \$1 is is 20 less than $\frac{5}{8}$ of the number of dollars in the bank? How many dollars had he in the bank?

24. What number is 25 smaller than 28?

25. What number is \$$\frac{5}{8}$ smaller than \$$\frac{2}{3}$?

26. What number is $\frac{2}{3}$ as large as $\frac{5}{8}$?

27. To what number must 11 be added to make $\frac{1}{2}$ of 24?

28. To what number must $\frac{5}{6}$ be added to make $\frac{9}{10}$?

29. 30 is the difference between 40 and what less number?

30. $\frac{3}{7}$ is the difference between $\frac{7}{9}$ and what less number?

31. To what number must $\frac{2}{3}$ of 5 be added to get $3\frac{1}{2}$ times 5?

32. What is the product of 25 and 6?

33. What is the product of $\frac{2}{3}$ and $\frac{7}{8}$?

34. What is 9 times 36? What are $\frac{2}{3}$ of 36?

35. What is $3\frac{2}{3}$ times $\frac{4}{5}$? $3\frac{2}{3}$ times $2\frac{2}{3}$?

36. What are $\frac{2}{3}$ of $4\frac{4}{5}$? Of $\frac{6}{7}$? Of 5? Of $5\frac{1}{3}$?

37. In what number is 5 contained 5 times?

38. In what number is $\frac{1}{3}$ of 5 contained 3 times? 5 times?

39. In $\frac{1}{3}$ of what number is $\frac{3}{4}$ of 6 contained $\frac{1}{3}$ of 5 times?

40. From what number can 7 be taken 8 times without leaving a remainder?

41. From $\frac{1}{2}$ of what number can 5 be taken 8 times without leaving a remainder?

42. From $\frac{2}{3}$ of what number can 7 be taken $3\frac{1}{3}$ times without leaving a remainder?

43. 5 is the sixth of what number?

44. $\frac{1}{3}$ is the sixth of what number?

45. 3 is $\frac{3}{7}$ of what number?

46. $\frac{3}{4}$ is $\frac{5}{8}$ of what number?

47. $1\frac{2}{3}$ is $\frac{6}{7}$ of 3 times what number?

48. $\frac{3}{8}$ is $\frac{4}{7}$ of $\frac{1}{3}$ of what number?

49. $\frac{5}{7}$ is $\frac{6}{7}$ of $\frac{3}{4}$ of what number?

50. $\frac{1}{4}$ of 7 is $\frac{6}{11}$ of $\frac{1}{4}$ of what number?

51. $\frac{3}{4}$ of $\frac{5}{8}$ is $\frac{1}{3}$ of $\frac{3}{5}$ of what number?

52. What number is $\frac{3}{4}$ of $2\frac{1}{2}$ times 25.

53. What number divided by 7 gives 12?

54. What number divided by $\frac{1}{3}$ gives 12?

55. What number divided by $\frac{3}{4}$ gives $\frac{2}{3}$ of 12?

56. What number contains 11 four times?

57. What number contains $\frac{1}{3}$, 5 times?

58. What number contains $\frac{2}{3}$ once? $\frac{1}{3}$ of a time?

59. What number contains $\frac{5}{8}$, $\frac{3}{4}$ of a time?

60. $\frac{3}{5}$ of a number is 5; what is the number?

61. $\frac{3}{4}$ of a number is $\frac{3}{7}$; what is the number?

62. From what number must $\frac{1}{5}$ of itself be taken to leave 8?

FORM.—*If* $\frac{1}{5}$ *of the number be taken from itself* $\frac{4}{5}$ *will remain; and if* 8 *is* $\frac{4}{5}$, $\frac{1}{5}$ *will equal* $\frac{1}{4}$ *of* 8 *or* 2, *and* $\frac{5}{5}$ *of the number will equal* 10.

63. From what number must $\frac{2}{5}$ of itself be taken to leave 12?

64. From what number must $\frac{3}{7}$ of itself be taken to leave $\frac{1}{7}$?

65. From what number must $\frac{5}{8}$ of itself be taken to leave $\frac{1}{3}$ of 8?

66. 36 is how many times 4?

67. 36 is how many times $\frac{1}{4}$?

68. 36 is how many times $\frac{2}{3}$?

69. How many times is $\frac{1}{4}$ contained in 36?

70. How many times are $\frac{3}{4}$ contained in 36?

71. How many times are $\frac{5}{6}$ contained in $\frac{1}{2}$ of 36?

72. How many times are $\frac{3}{4}$ contained in $\frac{7}{9}$ of 18?

73. What is $\frac{1}{5}$ of 90? $\frac{3}{5}$ of 90?

74. How many times 12 make 84?

75. How many times $\frac{3}{4}$ make 5?

76. How many times $\frac{5}{8}$ make $\frac{2}{3}$?

77. How many times $\frac{3}{7}$ make $\frac{5}{8}$ of 3?

78. How many times can 8 be taken from 48?

79. How many times can $\frac{2}{3}$ be taken from 20?

80. How many times can $\frac{3}{7}$ be subtracted from 4?

81. How many times can $\frac{2}{9}$ be subtracted from $\frac{5}{7}$?

82. The product of two numbers is 25; one of the numbers is 3; what is the other?

83. The product of 2 numbers is 37; one of the numbers is $\frac{3}{5}$; what is the other?

84. The product of 2 numbers is $\frac{5}{9}$; one of the numbers is $\frac{5}{6}$; what is the other?

85. The product of 2 numbers is $\frac{2}{7}$. one of the numbers $\frac{1}{8}$; what is the other?

86. Divide 8 into 7 equal parts; what is one of the parts?

87. Divide $\frac{2}{3}$ into 11 equal parts; what is one of the parts?

88. Divide $\frac{3}{4}$ into $\frac{2}{3}$ of 11 equal parts; what is one of the parts?

89. What number must be added 4 times to get 28?

FORM.—*If* 28 *is* 4 *times the number, the number must b* $\frac{1}{4}$ *of* 28, *which is* 7.

90. What number must be added $2\frac{1}{3}$ times to get 30?

91. What number must be added $5\frac{3}{8}$ times to get $\frac{1}{2}$?

92. What number must be multiplied by $\frac{5}{8}$ to get 80?

93. What number must be multiplied by $\frac{5}{7}$ to get $\frac{5}{8}$?

94. To what number must be added $\frac{5}{7}$ of itself to make 24?

FORM.—$\frac{5}{7}+\frac{7}{7}=\frac{12}{7}$; *If* 24 *is* $\frac{12}{7}$ *of the number,* $\frac{1}{7}$ *is* $\frac{1}{12}$ *of* 24, *which is* 2; *and* $\frac{7}{7}$ *is* 7 *times* 2, *which is* 14.

95. To what number must we add $\frac{3}{7}$ of itself to make $\frac{5}{8}$?

96. To what number must we add $\frac{1}{2}$ and $\frac{1}{3}$ of itself to make 22?

97. To make what number must we add $\frac{2}{3}$ and $\frac{3}{4}$ of itself to make $\frac{4}{11}$?

98. $\frac{1}{4}$ of a number taken from itself leaves $\frac{3}{5}$; what is the number?

99. $\frac{3}{4}$ of a number added to itself makes $\frac{3}{5}$: what is the number?

100. What will 5 bushels of potatoes cost at 25 cts. per bushel?

101. What will 3 barrels of flour cost at $\$12\frac{2}{3}$ a barrel?

102. What will $\frac{4}{5}$ of a bushel of wheat cost at $\$2\frac{1}{2}$ per bushel?

103. What will $\frac{1}{3}$ of a hundred-weight of coal cost at 75 cts. per cwt.?

104. What will $\frac{3}{5}$ of a hog weigh, if the whole hog weighs 500 lbs.

105. What will $\frac{3}{5}$ of a pie cost, if the whole pie costs 1 shilling?

106. What must I give for $\frac{3}{5}$ of a pound of sugar if 1 pound costs $\frac{1}{8}$ of a dollar?

107. If 40 bushels of wheat cost $80, what will 1 bushel cost?

108. If 20 bushels of apples cost $$30\frac{1}{3}$, what will 1 bushel cost?

109. If 6 lemons cost 18 $\frac{3}{5}$ cts., what will 1 cost?

110. If 5 apples are worth $\frac{1}{8}$ of a dollar, what is the value of one?

111. If $\frac{1}{2}$ of an orange is worth $\frac{1}{3}$ of a cent, what will 1 orange be worth?

112. If $\frac{3}{4}$ of a cheese is worth $3\frac{1}{8}$ dollars, what is the value of the whole cheese?

113. If $\frac{3}{7}$ of a pound of butter is worth $$\frac{4}{7}$, what is the value of 1 pound?

114. How many pounds of cheese at 7 cts., can be bought for 84 cts.?

115. How many quarts of vinegar at $\frac{1}{7}$ of a shilling can be bought for 12 shillings?

116. How many quarts of molasses at $\frac{3}{4}$ of a shilling a quart, can be bought for $5\frac{3}{7}$ shillings?

117. How many barrels of sugar at $$22\frac{3}{4}$ per barrel can be bought for $60?

118. How many apples at $\frac{3}{4}$ of a cent apiece can be bought for $\frac{5}{3}$ of a cent?

119. How many lemons at the rate of 7 for 8 cts., can be bought for 11 cts.?

120. How many bushels of corn at $$\frac{7}{2}$ a bushel can be bought for $\frac{2}{7}$ of a dollar?

121. I bought apples at the rate of 5 for 7 cts.; what did each cost?

122. I bought oranges at the rate of 11 for 13 cts.; I sold them at the rate of 13 for 11 cts.; did I make or lose, and how much on each?

123. If 9 men can do a piece of work in 3 days, how long will it take 1 man to do it?

124. If 6 men can do a piece of work in $\frac{5}{6}$ of a week, how long will it take 1 man to do it?

125. If 4 men can do $\frac{1}{2}$ of a piece of work in $\frac{3}{7}$ of a week, how long will it take 1 man to do the whole of it?

126. If 4 men can do $\frac{3}{4}$ of a piece of work in $\frac{5}{8}$ of a week, how long will it take 1 man to do it?

127. If 6 men can do a piece of work in 9 days, how many men would it take to do it in 1 day?

128. If 6 men can cut 12 cords of wood in 12 days, how many men would be required to cut it in 1 day?

129. How long will it take 1 man to do $\frac{2}{3}$ of what 9 men can do in $\frac{3}{5}$ of a day?

130. How many men would it take to build in $\frac{2}{3}$ of a day a barn that 7 men can build in $\frac{4}{15}$ of a month?

131. How long will a piece of work take one man, if 5 men can do it in $\frac{3}{5}$ of a week?

132. If a barrel of flour lasts a family of 6 persons 8 weeks, how long will it last 1 person?

133. If $\frac{2}{3}$ of a barrel of flour lasts a family of 3 persons $\frac{7}{8}$ of a month, how long will it last 1 person?

134. If 6 men can do a piece of work in 1 day, how many men will it take to do it in 3 days?

135. 6 men can do $\frac{3}{7}$ of a piece of work in $\frac{4}{7}$ of a day; how many men are required to do the whole of it in $\frac{2}{5}$ of a day?

ANAL. STEPS.—1. *Find the number of men necessary to do* $\frac{1}{7}$ *of the work.*

2. *Find the number of men necessary to do it in* $\frac{1}{7}$ *of a day.*

3. *Find the number of men necessary to do it in* $\frac{2}{5}$ *of a day.*

136. H can do a piece of work in 6 days; B can do the same work in 5 days; what part of the work can each do in 1 day?

137. D can do a piece of work in 3 days; E can do it in 5 days; what part of it can both do in 1 day.

138. A man can do a piece of work in 5 days; a boy can do $\frac{2}{3}$ as much as a man; what part of it can both do in a day, if they work together.

139. If 20 loaves last a family of 5 persons 6 days, how long will 5 loaves last a family of 3 persons?

FORM.—1. *If* 5 *persons require* 6 *days to consume the bread*, 1 *person will require* 5 *times* 6 *days, which is* 30 *days.*

2. *If* 20 *loaves require* 30 *days to consume it*, 1 *loaf will require* $\frac{1}{20}$ *of* 30 *days, which is* $\frac{3}{2}$ *of a day.*

3. *If* 1 *loaf requires* $\frac{3}{2}$ *of a day*, 5 *loaves will require* 5 *times* $\frac{3}{2}$, *or* $\frac{15}{2}$ *of a day.*

4. *If* 1 *person requires* $\frac{15}{2}$ *of a day to consume the bread*, 3 *persons will require* $\frac{1}{3}$ *of* $\frac{15}{2}$ *of a day, which is* $\frac{5}{2}$ *or* $2\frac{1}{2}$ *days.*

5. *Therefore, if* 20 *loaves last a family of* 5 *persons* 6 *days*, 5 *loaves will last a family of* 3 *persons* $2\frac{1}{2}$ *days.*

140. If 7 loaves of bread supply a family of 4 persons 4 days, how long will 8 loaves last a family of 10 persons?

141. If 16 horses eat a certain quantity of hay in $\frac{3}{7}$ of a week, how long will it take 11 horses to eat $\frac{2}{3}$ of the same quantity?

142. If $\frac{3}{5}$ of a barrel of flour lasts a family of 9 persons $\frac{3}{4}$ of a week, how much will be required to last a family of 11 persons $\frac{1}{2}$ of a week?

143. Two men travel in the same direction from the same place: one travels at the rate of 7 miles an hour; the other at the rate of 9 miles an hour: how far will they be apart at the end of 3 hours?

144. Two men start from places 3 miles apart, and travel in the same direction: one travels at the rate of 8 miles in 9 hours; the other follows at the rate of 9 miles in 8 hours: how far will they be apart in $7\frac{1}{2}$ hours?

145. Two men start from places 4 miles apart, and travel in opposite directions: one goes at the rate of 9 miles in 16

hours; and the other, at the rate of 7 miles in 15 hours: how far will they be apart in 3 hours?

146. A vessel sails for a port due east 3000 miles distant; after it has sailed 7 days at the rate of 70 miles in 6 hours, how far has it yet to sail before reaching its destination.

147. Norfolk, Va., is 2400 miles due west of Lisbon, in Portugal. A vessel from Lisbon sails for Norfolk at the rate of 60 miles per day. At the same time a vessel starts from Norfolk for Lisbon, and sails at the rate of 70 miles per day. When will they meet? How many miles apart will they be on the 25th day out?

148. 15 is what part of 30? ANS.—$\frac{15}{30}=\frac{1}{2}$.

149. 15 is how many times 30? ANS.—15 is $\frac{1}{2}$ *of once* 30.

150. 30 is what part of 15? ANS.—$\frac{30}{15}=2$.

151. 30 is how many times 15? ANS.—30 *is* 2 *times* 15.

152. 2 ounces is what part of 2 pounds?

153. A mill is what part of a cent? What part of a dollar?

154. $\frac{1}{5}$ of 10 is how many times 1?

155. $\frac{3}{7}$ of 21 is how many times 1?

156. $\frac{3}{4}$ of 7 is how many times 5?

FORM.—3 *times* $\frac{1}{4}$ *of* 7 *is* $\frac{21}{7}=3$, *and* 3 *is as many times* 5 *as* 5 *is contained in* 3, *which is* $\frac{3}{5}$.

157. $\frac{2}{3}$ of 5 is what part of 11?

FORM.—$\frac{2}{3}$ *of* 5 *is* $\frac{10}{3}$, *and* $\frac{10}{3}$ *is* $\frac{1}{3}$ *of* $\frac{10}{11}$ *of* 11, *or* $\frac{10}{33}$ *of* 11; *hence*, $\frac{2}{3}$ *of* 5 *is* $\frac{10}{33}$ *of* 11.

158. $\frac{3}{5}$ of 7 is what part of 12?

159. $\frac{6}{7}$ of 9 are how many parts of 1?

160. $\frac{1}{2}$ of 1 is what part of 8?

161. $\frac{3}{4}$ of 1 is what part of 9?

162. $\frac{6}{7}$ of 5 is what part of 4?

163. $\frac{3}{5}$ of 6 is what part of $\frac{1}{3}$ of 8?

164. $\frac{5}{7}$ is what part of $\frac{6}{11}$?

165. What is 3% of 18?

FORM.—3% of 18 *equals* 3 *times* $\frac{1}{100}$ *of* 18 which is $\frac{54}{100}$, or .54

166. What part of 1 is $12\frac{1}{2}\%$ of 18?

Form.—$12\frac{1}{2}\%$ *is* $\frac{25}{200}$, *or* $\frac{1}{8}$ *of* 18 ; $\frac{1}{8}$ *of* 18 *is* = $\frac{9}{4}$ *of* 1.

167. What is 3% of 9? Of 16? Of 24? Of 18?

168. How many hundredths in a number? Ans.—100 *hundredths.*

169. What is 100% of a number? Ans.—*The whole of the number.*

170. 8% is what part of any number?

Form.—8% *is* $\frac{8}{100}$ *of the number, and* $\frac{8}{100} = \frac{2}{25}$; *hence* 8% *of a number is* $\frac{2}{25}$ *of it.*

171. 5% is what part a number? 3%? 11%? $12\frac{1}{2}\%$? $23\frac{1}{3}\%$? $16\frac{2}{3}\%$?

172. $\frac{3}{5}$ of a number is what per cent. of it?

Form.—*Since* $\frac{5}{5}$ *of a number is* 100% *of it,* $\frac{3}{5}$ *of it is* $\frac{3}{5}$ *of* 100%, *which is* 60%.

173. $\frac{2}{3}$ of a number is what per cent. of it? $\frac{6}{7}$? $\frac{4}{5}$? $\frac{5}{7}$?

174. 6 is what per cent. of 7?

Form.—6 *is* $\frac{6}{7}$ *of* 7 ; $\frac{6}{7}$ *is* $85\frac{5}{7}\%$; *hence,* 6 *is* $85\frac{5}{7}\%$ *of* 7.

175. 3 is what per cent. of 9? Of 5? Of 2? Of $\frac{1}{2}$?

176. 5 is 3% of what number?

Form.—*If* 5 *is* 3%, 1% *is* $\frac{1}{3}$ *of* 5 *or* $\frac{5}{3}$, *and* 100% *is* 100 *times* $\frac{5}{3}$, *which is* $\frac{500}{3} = 166\frac{2}{3}$; *hence* 5 *is* 3% *of* $166\frac{2}{3}$

177. 4 is 6% of what number? 5%? 7%?

178. $\frac{2}{3}$ is 5% of what number?

179. $\frac{5}{7}$ is 2% of what number?

180. $\frac{2}{3}$ is what per cent. of $\frac{5}{6}$?

181. $\frac{5}{8}$ is what per cent. of $\frac{2}{3}$?

182. $\frac{1}{2}$ is 400% of what number?

183. The cost is 8 cts.; what is the gain if it is 5% of the cost? If 3%? If 9%? If $12\frac{1}{2}\%$? If $33\frac{1}{3}\%$? If 30%?

184. A gain of 8 cts. is 3% of the cost; what is the cost?

185. A gain of 7 cts. is 5% of the cost; what is the cost?

186. The cost is 8 cts.; the selling price is 12 cts.; the gain is what per cent. of the cost? What per cent. of the selling price? The selling price is what per cent. of the cost?

187. The cost is 12 cts., the gain is 4 cts.; the cost is what per cent. of the selling price? The gain is what per cent. of the selling price? The gain is what per cent. of the cost?

188. The selling price is 20 cts.; the cost is 20% of the selling price; what was gained? What per cent. of the cost was gained?

189. The gain is 60%; the selling price is 24 cts., what is the cost?

FORM.—*If the gain is* 60%, *the selling price must be* 160% *of the cost, and the cost must be* $\frac{100}{160}$, *or* $\frac{5}{8}$, *of the selling price.* $\frac{5}{8}$ *of* 24 *cts. equals* 15 *cts., the cost.*

190. The gain is 30%; the selling price is 5 cts.: what is the gain? The gain is what per cent. of the selling price?

191. The selling price is 50 cts., which is 75% of the cost; what per cent. of the selling price is the gain or loss?

192. The yearly interest at 6% is what part of the principal?

193. The interest for 2 years at 8% is what part of the principal?

194. The interest for 1 month at 6% is what part of the principal?

195. The interest for 3 months at 7% is what part of the principal?

196. The interest for 7 months at 8% is what part of the principal?

197. The interest for 1 day at 6% is what part of the principal?

198. The interest for 3 days at 8% is what part of the principal?

199. The interest for 8 days at 7% is what part of the principal?

200. If $36 is $\frac{4}{5}$ of the principal, what is the principal?

201. The amount of a note for 1 year is $20; the rate of interest is 8%; what is the principal?

202. What is the bank discount of $300 for 90 days at 7%? What are the proceeds?

203. What is the interest of $280 for 2 yr. 4 mo. at 8%? What is the amount?

204. What is the interest of $70 for 5 mo. 12 da. at 7%?

205. What is the interest of $25.25 at 7% for 36 days? For 12 days? For 90 days? For 60 days?

206. What is the interest on $380 at 6% for 33 days? For 93 days? For 63 days?

207. What principal at 7% in 1 year will give $55 interest? $36? $72?

208. What principal at 6% will yield $30 per month? $66? $70?

209. What principal at 6% will yield 30 cts. per day? 36? 25 cts.?

210. How long will it take $30 to double itself at 6%? At 7%? At 9%? At 8%?

211. I received $4 for the use of $20, 1 year; what is the rate per cent.?

212. I received $25 for the use of $90 for 4 months; what was the gain per cent.?

213. I have $2000 invested in the American Lock Co.; I receive a semi-annual dividend of $30: what rate of interest do I get on my investment?

214. A 8% semi-annual dividend is what per cent. per annum?

FORM.—*3% semi-annually is 6% per annum, but since 3% interest is paid at the end of 6 months it is worth 3% of the 3%, equal to* $\frac{8}{100}$ *of 1%, which added to 6% makes 6.09%.*

215. What is the annual worth of $800, for which I receive a semi-annual dividend of 3%? 2½%? 8%?

FORM.—*3% of $800 for 6 mos. is $24, and for 1 year it is $48. 3% of $24 for 6 mos. is 72 cts., which added to $48=$48.72, the annual worth.*

216. What is the annual worth of $1200 for which I receive a quarterly dividend of 2%? Of 3%? Of 5%?

217. Divide 45 into two parts that shall be to each other as 7 is to 8.

FORM.—1. *Since the numbers are to each other as 7 is to 8, hence 45 must be divided into 15 equal parts; 7 of these parts will equal one of the numbers, and 8 the other.* Or,

FORM.—2. *Since the numbers are to each other as 7 to 8, hence one number will be $\frac{7}{15}$ of 45, and the other $\frac{8}{15}$ of 45, equal to 21 and 24.*

218. Divide 24 into two numbers which shall be to each other as 6 and 7.

219. Divide 48 into 3 parts which shall be to each other as 3, 4, 5. As 6, 9, 9. As 6, 7, 11.

220. Two men bought a mill worth $5000. One paid $3 as often as the other paid $17. What part of the mill did each own? What is the value of each part?

221. Two men hired a pasture for $24; one put in 5 horses, the other 3 horses; what should each pay?

222. Charley had 9 cts. and Francis had 7 cts.; they gave them for 48 apples; how many should each have?

223. Henry and James bought a basket of apples, Henry paying 80 cts. and James 20 cts. They sold them so as to make 40 cts.; how should the money be divided?

224. A and B agree to do a job of work for $10. A quit when half done, and B finished it; how much should each receive?

FORM.—*If A and B working together do half the work, then A does ½ of ½ or ¼ of the whole, and B does ½ of ½+½ or ¾ of the whole; hence A is entitled to ¼ of $10, or $2½, and B is entitled to ¾ of $10, or $7½.*

225. Three men, A, B, and C, agree to build 50 rd. of stone wall for $100. After building 20 rd. A quit work; after building 40 rd. B quit, and C completed the job; how should the money be divided?

FORM.—*Since the first 20 rd. is,* $\frac{2}{5}$ *of the whole, the next 20 is* $\frac{2}{5}$ *of the whole, and the remainder is* $\frac{1}{5}$ *of the whole; then A, B, and C built* $\frac{2}{5}$ *of the whole, B and C built* $\frac{2}{5}$*, and C built* $\frac{1}{5}$*, hence,*

A built ($\frac{1}{3}$ *of* $\frac{2}{5}$)$=\frac{2}{15}$ *of the whole.*

B " ($\frac{1}{3}$ *of* $\frac{2}{5}$)$+$($\frac{1}{2}$ *of* $\frac{2}{5}$)$=\frac{1}{3}$ *of the whole.*

C " ($\frac{1}{3}$ *of* $\frac{2}{5}$)$+$($\frac{1}{2}$ *of* $\frac{2}{5}$)$+\frac{1}{5}=\frac{8}{15}$ *of the whole; hence A is entitled to* $\frac{2}{15}$ *of* $100, *or* 13\frac{1}{3}$; *B to* $\frac{1}{3}$ *of* $100, *or* 33\frac{1}{3}$; *and C to* $\frac{8}{15}$ *of* $100, *or* 53\frac{1}{3}$.

226. A, B, and C agree to build a house for $400. They do $\frac{1}{3}$ of the work, when A quits; after $\frac{2}{3}$ of the work is done B quits, and then C finishes the job; how should the money be divided?

227. A man agreed to cut 80 cords of wood for $60. After cutting 10 cords he took in a partner; after cutting 10 cords more they took in another partner to finish the job; how much should each receive?

228. A agreed to dig a well 40 feet for $100; but the last half is worth twice as much as the first half. After digging 20 feet he takes B as a partner; and A and B after digging to the depth of 30 feet take C as a partner *to finish*. How shall the money be divided?

229. A can do a piece of work in 5 days, and B in 3 days; in how many days can both do it by working together?

FORM.—*If A can do it in 5 days, he can do* $\frac{1}{5}$ *of it in 1 day; if B can do it in 3 days, he can do* $\frac{1}{3}$ *of it in 1 day, and both can do* $\frac{1}{5}+\frac{1}{3}$, *or* $\frac{8}{15}$ *of it in 1 day, and it would take them as many days as* $\frac{8}{15}$ *is contained times in* $\frac{15}{15}$, *which is* 1$\frac{7}{8}$ *days.*

230. A can do a piece of work in 5 days; B can do it in 2 days; and C can do it in 6 days: how many days will it take if all work together?

231. A can build a barn in 40 days; B can build it in 50 days; and C can build it in 60 days. A and B work together until it is half finished, when C helps to finish it. They receive $740 for the work. How shall the money be divided?

232. There are 3 pipes to a cistern, the first can empty it in 5 hours, the second can empty it in 6 hours, and the third can empty it in 8 hours; how many hours will it take for all to empty it?

233. Five men agree to do a piece of work for $60, each to receive an equal part; when the work is ¼ done, 4 of the men quit, and the fifth man finishes it; how much should each receive?

234. A can build a wall in 10 days, and B can do it in 15 days; after working 3 days together, B leaves A to finish it; how long will it take him?

LESSON LXXIII.

METRIC SYSTEM OF WEIGHTS AND MEASURES.

66. The Metric System of weights and measures is formed upon the decimal scale, and has for its base an invariable unit derived from nature, and called a METRE; and upon this unit all the units of weight and measure are based.

67. The Metre is the *ten-millionth* part of the distance from the equator to the pole; and is the principal unit of Linear measure.

68. The Are is a square whose side is ten *metres*. It is the principal unit of superficial measure.

69. The Stere is a cube whose edge is a *metre*. It is the principal unit of solid or cubic measure.

70. The Litre is a cube whose edge is the *tenth* of a *metre*. It is the principal unit of all *measures of capacity*.

71. The Gram is the weight of a cube of pure water at its *greatest density*, whose edge is the hundredth part of a metre.

A litre of water weighs 1,000 grams. It is the principal unit of weight.

72. The names of the derivative denominations are formed by joining a Latin or Greek prefix to the principal units. There are seven of these prefixes derived as follows:

Latin.	MILLI,	from *Millesimus*,	*a thousandth.*
	CENTI,	from *Centesimus*,	*a hundredth.*
	DECI,	from *Decimus*,	*a tenth.*
Greek.	DECA,	*ten.*	
	HECTO,	from *Hecaton*,	*one hundred.*
	KILO,	from *Kilioi*,	*one thousand.*
	MYRIA,	*ten thousand.*	

The formation of the tables can be seen at a glance by the following:

$$\left.\begin{array}{l}\textit{Milli}\\ \textit{Centi}\\ \textit{Deci}\\ \hline \textit{Deca}\\ \textit{Hecto}\\ \textit{Kilo}\\ \textit{Myria}\end{array}\right\}\text{METRE.}\left.\vphantom{\begin{array}{l}1\\1\\1\\1\\1\\1\\1\end{array}}\right\}\text{ARE.*}\left.\vphantom{\begin{array}{l}1\\1\\1\\1\\1\\1\\1\end{array}}\right\}\text{STERE.}\left.\vphantom{\begin{array}{l}1\\1\\1\\1\\1\\1\\1\end{array}}\right\}\text{LITRE.}\left.\vphantom{\begin{array}{l}1\\1\\1\\1\\1\\1\\1\end{array}}\right\}\text{GRAM.}$$

NAMES.	PRONUNCIATION.	ABR.	NAMES.	PRONUNCIATION.	ABR.
Millimetre	Mill'-e-mee'-ter	*mm.*	Hectostere	Hec'-to-stêr	*hs.*
Centimetre	Sent'-e-mee'-ter	*cm.*	Kilostere	Kill'-o-stêr	*ks.*
Decimetre	Des'-e-mee'-ter	*dm.*	Myriastere	Mir'-e-a-stêr	*mys.*
Metre	Mee'-ter	*m.*	Millilitre	Mill'-e-li'-ter	*ml.*
Decametre	Dek'-a-mee-ter	*dkm.*	Centilitre	Sent'-e-li'-ter	*cl.*
Hectometre	Hec'-to-mee-ter	*hm.*	Decilitre	Des'-e-li'-ter	*dl.*
Kilometre	Kill'-o-mee-ter	*km.*	*Litre*	Li'-ter	*l.*
Myriametre	Mir'-e-a-mee'-ter	*mym.*	Decalitre	Dek'-a-li-ter	*dkl.*
Milliare	Mill'-e-âre	*ma.*	Hectolitre	Hec'-to-li'-ter	*hl.*
Centiare	Sent'-e-âre	*ca.*	Kilolitre	Kill'-o-li'-ter	*kl.*
Deciare	Des'-e-âre	*da.*	Myrialitre	Mir'-e-a-li'-ter	*myl.*
Are	Are	*a.*	Milligram	Mill'-e-gram	*mg.*
Decare	Dek'-âre	*dka.*	Centigram	Sent'-e-gram	*cg.*
Hectare	Hec'-târe	*ha.*	Decigram	Des'-e-gram	*dg.*
Kilare	Kill'-âre	*ka.*	*Gram*	Gram	*g.*
Myriare	Mir'-e-âre	*mya.*	Decagram	Dek'-a-gram	*dkg.*
Millistere	Mill'-e-stêr	*ms.*	Hectogram	Hec'-to-gram	*hg.*
Centistere	Sent'-e-stêr	*cs.*	Kilogram	Kill'-o-gram	*kg.*
Decistere	Des'-e-stêr	*ds.*	Myriagram	Mir'-e-a-gram	*myg.*
Stère	Stêre	*s.*	Quintal	Quin'-tal	*q.*
Decastere	Dek'-a-stêr	*dks.*	Tonneau	Tun'-no	*T.*

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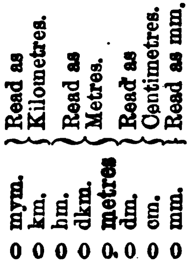
* *The a in deca and myria, and the o in hecto and kilo are dropped when prefixed to Are.*

SCALES AND TABLES.

(*a.*) SCALE.



1 Metre = 3 ft. 3⅜ in. (nearly).
1 Kilometre = 198 rd. 13 ft. 10 in.

(*b.*) TABLE.*

10 mm. = 1 cm.
10 cm. = 1 dm.
10 dm. = 1 *m.*
10 m. = 1 dkm.
10 dkm. = 1 hm.
10 hm. = 1 km.
10 km. = 1 mym.

(*Contracted.*)

10 mm. = 1 cm.
100 cm. = 1 m.
100 m. = 1 km.

73. The Centimetre is the unit generally used for measurements less than a metre.

74. The Metre is the unit commonly used by artizans.

75. The Kilometre is the unit commonly used by surveyors in measuring distances.

1. How many centimetres in 3 metres? In 12 metres?
2. How many metres in 36 kilometres? In 11 kilometres?
3. How many millimetres in 11 metres? In 16 metres?
4. How many centimetres in 5 kilometres? In 7 kilometres?
5. How many centimetres in 400 millimetres? In 1600 millimetres?
6. How many metres in 3467 centimetres? *Ans.* 34.67 *m.*
7. How many kilometres in 374 metres? In 3654 centimetres?

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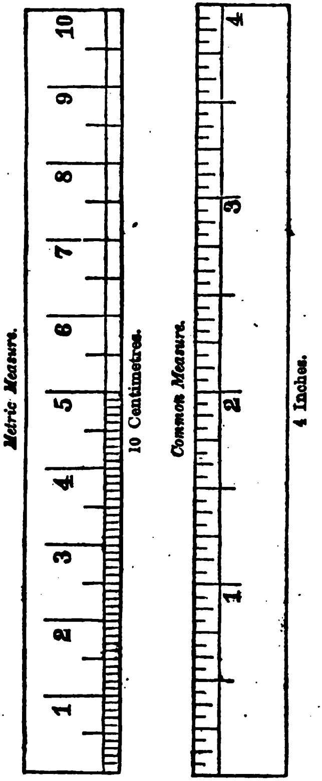
* The table for each unit is formed in the same manner. For the remaining units only contracted tables will be given, for the intermediate denominations being very seldom used in practice, it is worse than useless to thrust upon the pupil's attention such a multiplicity of names.



NOTE.—By the accompanying illustration it will be seen that *one-tenth* of a metre, or ten centimetres equals about 3$\frac{15}{16}$ in., or a trifle short of 4 in.

This measure as well as the other measures and weights, are written as whole numbers and decimals. The decimal point is placed at the right of the *unit;* thus, 4.167 *m.* may be written 416.7 *cm.* To make a metric rule, cut a piece of wood, paper, or tape, 39$\frac{3}{8}$ in. long. Divide it into 10 equal parts, and each part into ten *other* equal parts; each of these parts is 1 *centimetre.* Divide each centimetre into ten equal parts, and each part is a *millimetre.*

The diameter of the nickel five cent piece of 1866 is 2 *centimetres*, and its weight is 5 *grams.*

REDUCTION, ADDITION, AND SUBTRACTION.

To reduce to lower denominations.

Reduce 37.04 *km.* to metres.

OPERATION.—37.04 *km.* × 1000=37040 m. *Ans.*

FORM.—Since in 1 *km.* there are 1000 *m.*, in 37.46 *km.* there are *37.46 times 1000 m.*, which is 37460 *m.*; therefore the following (*a.*) **Rule.**—*Multiply the given denomination by* 10, 100, 1000, *&c., as the case may require, by removing the decimal point to the right.*

To reduce to higher denominations.

Reduce 4130.3 metres to kilometres.

OPERATION.—4130.3 *m.*÷1000=4.1303 *km.* *Ans.*

FORM.—Since in 1000 metres there is one kilometre, in 4130.3 *m.* there are as many kilometres as 1000 *m.* are contained times in 4130.3 *m.*, which are 4.1303 *km.*; hence the following

(*b.*) **Rule.**—*Divide the given denomination by* 10, 100, 1000, *&c., as the case may require, by removing the decimal point to the left.*

To add or to subtract the metric weights and measures, is given the following

(*c.*) **Rule.**—*Change the given numbers to the same denomination, then add or subtract as in decimal fractions.*

1. Bought 3 pieces of tape, the first contained 15 *m.*, the second 14 *m.*, and the third 7 *m.*; how much was bought in all?

2. Sold 5.5 *m.* of calico and 7.75 *m.* of muslin; how many metres did I sell in all?

3. John is 2.05 *m.* high, and Henry is 1.75 *m.* high; how much taller is John than Henry?

4. How many metres in one kilometre? In 3 kilometres?

5. How many metres in 3.5 kilometres? In 4.3 kilometres?

6. Add the following in metres:

4.32 *km.* = 4320. *m.*
46.07 *km.* = 46070. *m.*
27.371 *km.* = 27371. *m.*
4170 *m.* = 4170. *m.*
24.67 *m.* = 24.67 *m.*
81955.67 *m.*

(7.)	(8.)
43.37 *km.*	3.47 *m.*
20.3 *km.*	6743 *m.*
40.6 *m.*	8637 *km.*
.876 *m.*	86.85 *km.*
4.67 *km.*	4.96 *m.*

Reduce and add the following in kilometres:

(9.)	(10.)	(11.)	(12.)	(13.)
41.37 *m.*	8.34 *m.*	41.3 *km.*	41.8 *km.*	673. *km.*
20 *m.*	7.63 *km.*	31.67 *m.*	8676 *m.*	6.03 *km.*
67.3 *km.*	417 *km.*	46.7 *m.*	41.36 *m.*	3067 *m.*
467 *m.*	38.67 *m.*	867 *km.*	43.07 *m.*	4136 *m.*
8967 *m.*	71.3 *m.*	.413 *m.*	413.6 *m.*	8913 *m.*

14. From 436 *km.* Take 673 *m.*

15. From 41367 *m.* Take 25 *km.*

16. From 46 *km.* Take 387 *m.*

17. What will 3.5 *m.* of calico cost at 25 cts. per metre?

18. How many dresses containing 4.4 *m.*, can be cut from a piece containing 25.34 *m.*? What will remain?

MODEL OPERATION.

(*a.*)	(*b.*)
3.5 *m.*	*m.* *m.*
25 cts.	4.4)25.34(5 *Ans.*
——	22.0
17.5	——
70	3.34 *m.* Rem.
——	
87.5=87½ cts.	

Form.—(*a.*) *Since* 1 *metre costs* 25 *cts.*, 3.5 *m. will cost* 3.5 *times* 25 *cts.*, *which are* 87.5 *cts.*, *equal* 87½ *cts.*

Form.—(*b.*) *Since* 4.4 *m. make* 1 *dress, there can be as many dresses cut from a piece containing* 25.34 *m.*, *as* 4.4 *m. are contained times in* 25.34 *m.*, *which are* 5 *times with a remainder*, 3.34 *m.*

19. Bought 3.47 *m.* of silk at 3 cts. per centimetre; what did it cost?

20. A man bought 3.76 metres of cloth and sold 3% of it; how much remained?

21. Bought silk at $3 per metre, and sold it at an advance of 20%; what did I get for it?

22. If I cut 63 centimetres of ribbon from a piece containing 7.34 *m.*; what will the remainder be worth at 50 cts. per metre?

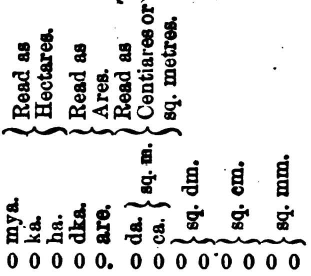
SQUARE MEASURE.

76. The **Square Metre** is the unit commonly used by artisans in specifying surfaces of small extent. It contains about 10 sq. ft., 110 sq. in.

77. The Are is the unit commonly used to express quantities less than the hectare. 100 Ares make one hectare.

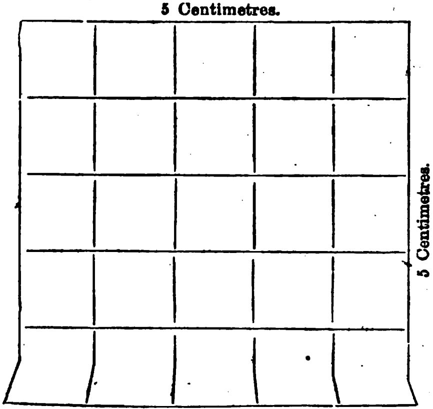
78. The Hectare is the unit commonly used by surveyors in estimating the contents of land. It contains 2.471 acres. 100 ares make 1 hectare.

(*a.*) SCALE.



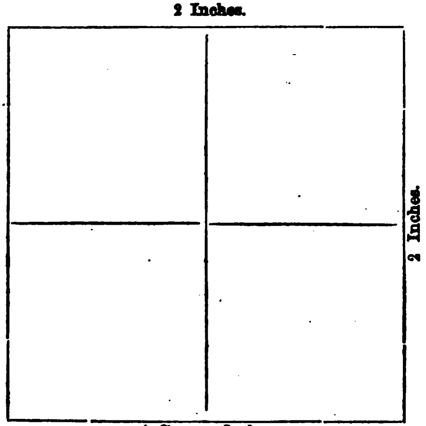
(*b.*) TABLE.

100 sq. millimetres = 1 sq. cm.
100 sq. centimetres = 1 sq. dm.
100 sq. decimetres = $\left\{ \begin{array}{l} \text{1 sq. m.} \\ \text{1 ca.} \end{array} \right.$
$\left. \begin{array}{l} \text{100 sq. metres} \\ \text{100 centiares} \end{array} \right\}$ = 1 are.
100 ares = 1 ha.



25 Square Centimetres.

NOTE.—It will be seen by the accompanying illustration that 25 centimetres contain an area a trifle less than 4 sq. inches. It will also be seen that the superficial contents is found by *multiplying the length by the breadth;* hence a square 10 centimetres long and 10 centimetres wide, or 100 sq. centimetres make 1 sq. decimetre, and for the same reason 100 sq. decimetres make 1 sq. metre or *centiare.*

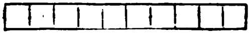


4 *Square Inches.*

ILLUSTRATIONS.

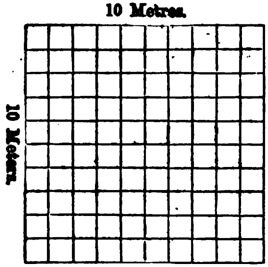
1 *Sq. metre.*

1 sq. metre=1 centiare.



1 *Deciare.*

$$\left.\begin{array}{l}\text{10 sq. metres}\\ \text{10 centiares}\end{array}\right\}\text{1 deciare.}$$



1 *Are.*

$$\left.\begin{array}{l}\text{100 sq. metres}\\ \text{100 centiares}\\ \text{10 deciares}\end{array}\right\}\text{=1 are.}$$

NOTE.—The formation of the *are* will readily be seen by the above illustrations, and in the same manner the *decare* and *hectare* are found. The *hectare* is a square containing 10,000 *square metres.*

23. How many sq. metres in the walls of a room 3 metres high, 8 metres long, and 7 metres wide?

24. How many sq. metres in the ceiling of the same room?

25. How many sq. metres in the walls and ceiling of a room 2.75 metres high, 8.5 metres long, and 6 metres wide?

26. How much will it cost to plaster the walls and ceiling of a room 8.3 metres long, 7 metres wide, and two metres high, at $1.75 per sq. metre?

27. How many square metres of boards will it take to make a box with lid 1.5 metres long, 1.3 metres wide, and 72 centimetres deep, making no allowance for thickness?

28. In 3742.74 sq. metres how many hectares?

29. The length of a certain field is 146 *m.*, the width is 125 *m.* 36 *cm.*; how many hectares does it contain?

30. I bought a tract of land 6 *km.* long and 2.57 *km.* wide; what was it worth at $3.40 per hectare?

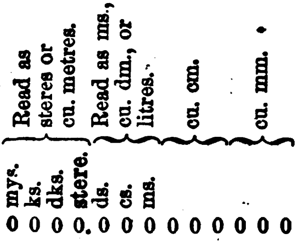
31. The width of a piece of land is 300 metres, what must be the length to contain 12.24 hectares?

32. The length of a board is 47 centimetres, what must be the width to contain 4,000 sq. centimetres?

CUBIC MEASURE.

79. The Cubic Metre or Stere is the unit commonly used by engineers in estimating the solid contents of embankments, cellars, walls, etc. It equals 1.308 cu. yards.

(*a.*) SCALE.



(*b.*) TABLE.

1000 cu. mm. = 1 cu. cm.

1000 cu. cm. = { 1 cu. dm. / 1 litre. / 1 ms.

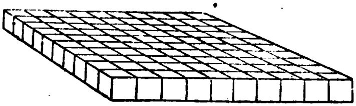
1000 cu. dm. = { 1 cu. m. / 1 stere.

1000 steres = 1 ks.

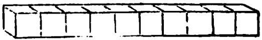
ILLUSTRATIONS.

1 *Millistere.*

$$\left.\begin{array}{l}\text{1 cu. decimetre}\\ \text{1 litre}\end{array}\right\}=\text{1 millistere.}$$



1 *Decistere.*

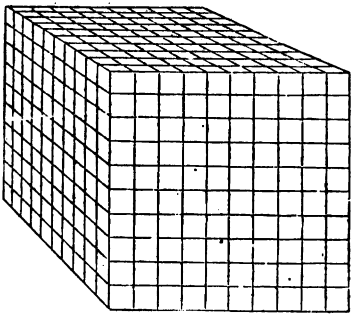


1 *Centistere.*

$$\left.\begin{array}{l}\text{100 litres}\\ \text{100 cu. decimetres}\\ \text{100 millisteres}\\ \text{10 centisteres}\end{array}\right\}=\left\{\begin{array}{l}\text{1 hectolitre.}\\ \text{1 decistere.}\end{array}\right.$$

$$\left.\begin{array}{l}\text{10 millisteres}\\ \text{10 c. decimetres}\\ \text{10 litres}\end{array}\right\}=\left\{\begin{array}{l}\text{1 centistere.}\\ \text{1 decalitre.}\end{array}\right.$$

$$\left.\begin{array}{l}\text{1000 litres}\\ \text{1000 c. decimetres}\\ \text{1000 millisteres}\\ \text{100 centisteres}\\ \text{10 decisteres}\end{array}\right\}=\left\{\begin{array}{l}\text{1 c. metre.}\\ \text{1 kilolitre.}\\ \text{1 stere.}\end{array}\right.$$



1 *Stere.*

NOTE.—By the above illustrations will be seen the formation of the *stere* or *cubic metre.* In the same manner the kilostere may be found, which is a cube whose edge is 10 metres, and contains 1,000 cu. metres, or 1,000,000 litres.

33. How many cubic metres in a box 3 *m.* deep, 4 *m.* long, 25 *cm.* wide?

34. I have a pile of wood 36.7 metres long, 1.3 wide, and 1.8 *m* high; what is it worth at 27 cts. per stere?

35. How deep must be a box 2.25 *m.* square, to contain 9.8 steres?

36. How many steres in a wall 25 *m.* long, 3.4 *m.* high, and 24.3 *cm.* thick?

37. What must I give to remove the earth from a cellar 3.2 *m.* deep, 9 *m.* long, and 7.5 *m.* wide, at 56 cts. per stere?

38. I wish a square box which will contain 1,000 litres of corn; what are its dimensions?

39. How many litres of wheat can be put in a box 3.4 *m.* long, 3.5 *m.* deep, and 2.1 *m.* wide?

DRY AND LIQUID MEASURE.

30. The Unit commonly used in the measurement of grain, roots, and liquids by the barrel, is the hectolitre. It equals 26.417 gal. wine measure, or 2.839 bu. dry measure.

31. The Unit commonly used by grocers is the *litre.* It equals 1.0567 qt. wine measure, or .908 qt. dry measure, or a trifle more than a wine quart.

(*a.*) SCALE.

Read as hectolitres.			Read as l. or cu. dm.		Read as centilitres.		
myl.	kl.	hl.	dkl.	litre.	dl.	cl.	ml.
0	0	0	0	0.	0	0	0

(*b.*) TABLE.

10 millilitres = 1 cl.

100 centilitres = { 1 litre. 1 cu. dm.

100 litres = 1 hl.

1000 litres = { 1 kl. 1 cu. m.

32. To find the contents of bins or cisterns in litres.

Rule.—*The product of the length, breadth, and thickness, in decimetres, will give the contents in litres.*

40. How many hectolitres of wine in a tank 4.3 *m.* deep, 5 *m.* wide, and 6 *m.* long?

41. What will be the value of the wheat in a bin 4.8 *m.* deep, 2.4 *m.* long, and 1.8 *m.* wide, at $4.30 per hectolitre?

42. What is the depth of a box 6 *m.* long and 5 *m.* wide, that will hold 1.250 hectolitres of corn?

43. I wish to build a cistern that will hold 2.025 hectolitres of water; how deep must it be if the top is 4.5 *m.* square?

44. A man bought 4.3 litres of beans; what was the cost at 37½ cts. per litre?

45. 5.3 litres of potatoes cost $1.30; how much is that per litre?

46. Bought 1.8 hectolitre of molasses for $35; how much was that per litre?

47. What must I give for 13.2 litres of vinegar, at 8 cts. per litre?

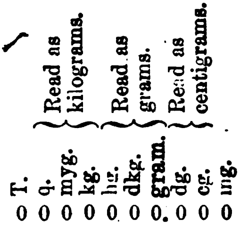
WEIGHT.

83. The Unit commonly used in philosophical experiments, by jewellers and druggists, is the *gram*. Its weight is 15.432 gr. Troy.

84. The Unit commonly used by grocers is the *kilogram*, commonly contracted to *kilo*. It is the weight of a litre of pure water, and equals 2.2046 lb., or about 2⅕ lb. avoirdupois.

85. The Unit commonly used in weighing heavy bodies, as coal, iron, marble, R. R. freight, &c., is the *Tonneau*. It is the weight of a cubic metre of pure water, and equals 2204.6 lb. avoirdupois.

(*a.*) SCALE.



(*b.*) TABLE.

10 milligrams = 1 cg.

100 centigrams = 1 gram.

1000 grams = 1 kg.

1000 kilograms = 1 T.

86. To find the weight of water is given the following

Rule.—(*a.*) *The contents in cu. centimetres equals the number of grams.*

NOTE.—A cu. metre of water weighs 1 tonneau, and a litre of water weighs 1 kilogram.

48. What is the weight of water contained in a tank 4.3 *m.* long, 3 *m.* wide, and 2.5 *m.* deep?

49. How many tonneaux of water in a dock 125 *m.* long, 75 *m.* wide, and 10 *m.* deep?

50. There is a tank 12 *m.* square; what is the weight of a vessel that causes the water to rise in it 2.5 metres?

51. What is the cost of 35 centigrams of tea, at $4.60 per kilogram?

52. I bought 37 kilos of sugar, at 15 cts. per kilo; what was the amount of the bill?

53. At $15 per tonneau, what is that per kilo?

54. If ten kilos of coffee cost $1.38, how much is that per kilo?

55. I bought 3.4 kilos of indigo at $8 per kilo; 4.7 kilos of logwood at 16 cts. per kilo; what was the amount of the bill?

56. A man bought a gold chain weighing 13.7 grams, at 16 cts. per gram; what was the cost?

57. I paid $3.40 for a chain weighing 8.7 grams; how much was that per gram?

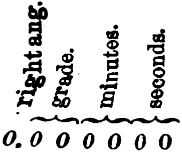
58. I wish to make 15 grams of quinine into 11 doses; how much in each dose?

59. How many pills of equal size can be made from 14 grams of ipecac, 11 grams of jalap, and 5 grams of liquorice, each pill to contain 35 centigrams?

MEASUREMENT OF ANGLES.

87. In the Centesimal or French method, the *right angle* is divided into 100 equal parts called *grades*, the grade into 100 equal parts called *minutes*, the minute into 100 equal parts called *seconds*.

(*a.*) SCALE.



(*b.*) TABLE.

100 seconds (") = 1 minute (').

100 minutes = 1 grade (gr).

100 grades = 1 right angle (r. a.).

NOTE.—Since the signs for both the common and centesimal methods are the same, to prevent confusion when minutes and seconds are expressed in the centesimal method, annex the abreviation, *cen.;* thus, 3' 46' *cen.*

60. How many minutes in 3 r. a.? In 360 degrees?

61. The sum of two angles is what, when one measures 46' 37" *cen.*, and the other 23' 86" *cen.*?

62. Add 4 gr. 40" *cen.*, 2' 4" *cen.*, 78" *cen.*, 4 gr. 3' 46".

63. The sum of two angles is 3 gr. 4", one is 1 gr. 5'; what is the other?

88. *To reduce the common to the metric system.*

LINEAR MEASURE.

(*a.*) *Table* of equivalents.*

1 in. = 25½ mm. (nearly).	1 mi. = 1609.35 *m.*
1 ft. = 305 mm. (nearly).	1 cm. = .3937 = ⅜ in. (nearly).
1 yd. = 914 mm.	1 m. = 39.37 in. = 1.093 yd.
1 rd. = 5029 mm.	1 km. = .62137 mi. = 198 rd. 13 ft, 10 in.

(*b.*) SQUARE MEASURE.—*Table.*

1 sq. in. = 6.5 sq. cm.	1 sq. cm. = .155 sq. in.
1 sq. ft. = 9.3 sq. dm.	1 sq. m. = { 1550 sq. in. 1.076 sq. ft.
1 sq. yd. = .835 sq. m.	1 are. = 119.6 sq. yd.
1 acre = 40.47 a.	1 ha. = 2.471 acres.

(*c.*) CUBIC MEASURE.—*Table.*

1 cu. in. = 16.387 cu. centm.	1 litre = { 1.0567 qt. liq. meas. .908 qt. dry meas.
1 cu. ft. = { 28.34 litres. .0283 steres.	1 hectolitre = { 2.837 bu. dry meas. 26.417 gal. liq. meas.
1 cu. yd. = .76531 steres.	1 kilolitre 1 cu. metre 1 stere } = { 35.316 cu. ft. 1.308 cu. yd. 264.17 gal. liq. meas. .2759 cord.
1 cord = 3.6281 steres.	
1 fluid oz. = .02958 litres.	
1 gal. = 3.786 litres.	
1 bus. = 35.24 litres.	

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* Authorized by Act of Congress, July 27, 1866.

(d.) WEIGHT—*Table.*

1 oz. troy = 31.1 grams.
1 lb. troy, 1 lb. apoth. = 373.2 "
1 oz. avoir. = 28.35 "
1 lb. avoir. = 453.6 "

1 ton avoir. = 907.2 kilos.
1 gram = 15.432 gr. troy. / .5643 dr. avoir.
1 kilogram = 2.2046 lb. avoir.
1 tonneau = 2204.6 lb. avoir.

(e.) ANGULAR MEASURE.—*Table.*

1 r. a. = 100 grades.
1° = 1$\frac{1}{9}$ grades.
1' = 1.85 minutes ('cen.)
1'' = 3.08 seconds (''cen.)

1 cir. = 400 grades.
1 grade = 9 deg.
1' cen. = 5.4'
1'' cen. = 3.24''.

Reduce 8 ft. 9 in. to the metric system.
Reduce 30 bu. 3 pks to the metric system.

MODEL OPERATION.

(a.)

$$\overset{mm.}{305} \times \overset{ft.}{8} = 2.44 \text{ m.}$$

$$25\tfrac{1}{2} \times \overset{in.}{9} = .229 \text{ ``}$$

$$2.669 \text{ ``}$$

(b.)

$$\overset{l.}{4)35.24} \times \overset{bu.}{30} = 10.57 \text{ hl.}$$

$$8.81 \times \overset{pks.}{3} = .26 \text{ ``}$$

$$10.83 \text{ `` } \textit{Ans.}$$

FORM.—(*a*). 1. *Since in* 1 *ft. there are* 305 *mm., in* 8 *ft. there are* 8 *times* 305 *mm., which are* 2.44 *m.*

2. *Since in* 1 *inch there are* 25½ *mm. in* 9 *inches there are* 9 *times* 25½ *mm., which are* .229 *m.*; *and the sum of* 2.44 *m. and* .229 *m. equals* 2.669 *m., therefore* 8 *ft.* 9 *in. equals* 2.669 *metres.*

ANAL. STEPS.—1. *Find the number of hectolitres in* 30 *bushels.*
2. *Find the number of hectolitres in* 1 *peck, then* 3 *pecks.*
3. *Find the sum of hectolitres in all.*

(*c.*) **Rule.**—*Multiply the number to be reduced by the value of* ONE *of its units in the* METRIC SYSTEM.

64. Reduce 6 mi. 13 rds. to kilometres.
65. Reduce 1 yd. 2 ft. 5 in. to metres.
66. Reduce 5 in. to hectares.
67. Reduce 1 sq. yd. 4 sq. ft. to ares.
68. Reduce 2 sq. ft. 17 sq. in. to centiares.
69. Reduce 46 sq. ft. 13 sq. in. to sq. metres.
70. Reduce 7 cords to steres.
71. Reduce 12 cu. yd. 3 cu. ft to steres.
72. Reduce 14 bu. 3 pks. to litres.
73. Reduce 11 cu. ft. 7 cu. in. to litres.
74. Reduce 3 cu. yd. 4 cu. ft. to hectolitres.
75. Reduce 4 lb. 3 oz. avoirdupois to kilograms.
76. Reduce 5 lb. 6 oz. troy to grams.
77. Reduce 63 gal. 2 qts. to hectolitres.
78. Reduce 3° 4' 36" to the centesimal scale.
79. Reduce 17° 5' 40" to the centesimal scale.
80. Reduce 3 bar. 14 gal. to hectolitres.

89. *To reduce the metre to the common system.*

Reduce 43.6 metres to yards.

Reduce 46.3 *hl.* to liquid measure.

MODEL OPERATIONS.

(*a.*) $\overset{\text{m.}}{43.6} \times \overset{\text{yd.}}{1.093}$ = 47 yd. 1 ft. 9.6 in. *Ans.*

(*b.*) $\overset{\text{hl.}}{46.3} \times \overset{\text{gal.}}{26.417}$ = 19 bar. 26 gal. 3.2 gi. *Ans.*

FORM.—(*a.*) *Since in* 1 *metre there are* 1.093 *yd. in* 43.6 *m., there are* 43.6 *times* 1.093 *yd., which are* 47.6 *yds., equal to* 47 *yd.* 1 *ft.* 9.6 *in.*

FORM.—(*b.*) *Since in* 1 *hectolitre there are* 26.417 *gal. in* 46.3 *hectolitres there are* 46.3 *times* 26.417 *gal., which are* 1223.1 *gal., equal to* 19 *bar.* 26 *gal.* 3.2 *gills ; hence the following*

(*c.*) **Rule.**—*Multiply the number to be reduced by the value of* ONE *of its units in the* COMMON SYSTEM.

81. Reduce 41.7 kilometres to miles.

82. Reduce 37.4 sq. metres to sq. ft.

83. Reduce 47.3 hectares to acres.

84. Reduce 43.3 litres to cu. ft.

85. Reduce 43 steres to cords.

86. Reduce 3.41 cu. metres to cu. ft.

87. What will a piece of cloth containing 4.36 *m.* cost at $2.50 per yd.? *Ans.* $118.

88. What will 12 yd. of broadcloth cost at $4.50 per metre? *Ans.* $53.46.

89. Bought 30 pieces of calico, each containing 23.64 metre at 60 cts. per metre; what was the amount of the bill?

90. Sold 23.4 *m.* of broadcloth, which cost 80 cts.; at an advance of 20% what did I make per metre?

91. What is the cost of putting up 684.37 *km.* of telegraph wire at $1.50 per kilometre?

92. How much will it cost to transport a car load of freight 826 *km.* at 10 cts. per mile?

93. What must be the length of a board 30 centimetres wide to contain 5 sq. feet? *Ans.* 1.55 metres.

94. What is the value of a lot 126×28 ft. at $3 per sq. metre? *Ans.* $984.312.

95. How many sq. metres in the walls of a room 3 metres high, 8 metres long, and 7 metres wide?

96. How many metres in the ceiling of the same room?

97. How many sq. metres in the walls and ceiling of a room 2.75 metres high, 8.5 metres long, and 6 metres wide?

98. How much will it cost to plaster the walls and ceiling of a room 8.3 metres long, 7 metres wide, and 2 metres high, at $1.75 per sq. metre?

99. How many boards will it take to make a box with lid, 1.5 metres long, 1.3 metres wide, and 7.2 metres deep, making *no* allowance for thickness?

100. How many hectares in a lot of land 437.3 metres long *and 403.4 metres* wide. How many acres?

TEST QUESTIONS FOR EXAMINATION.

66. How is the metric system of weights and measures formed?

67. What is the length of a metre? Of what is it the principal unit?

68. What is an are? What are its dimensions? Of what is it the principal unit?

69. What is the principal unit of cubic measure?

70. What are the dimensions of the litre? What is its use?

71. What is the use of a gram? How is the weight of a gram determined?

72. How are the names of the derivative denominations formed? How many prefixes are used? From what are they derived? Recite the contracted table of linear measure.

73. What unit is generally used for measurements less than a metre?

74. What unit is commonly used by artisans?

75. What unit is commonly used by surveyors? What is the length of 10 centimetres? How may a metric rule be made? What is the rule for reducing to lower denominations? What is the rule for reducing to higher denominations?

76. What is the use of the square metre? What is its area?

77. For what is the are used? How many ares make a hectare?

78. What is the unit used by surveyors? How many acres does it contain? About how many square inches are in 25 sq. centimetres? Recite the table.

79. What is the use of the cubic metre or stere? How many cu. yards does it contain? Illustrate the formation of the declare. The formation of the are. The formation of the centistere. What denominations are equivalent to the millistere? Show the formation of the centistere. Show the formation of the decistere. The formation of the stere. Recite the table.

80. What unit is commonly used in the measurement of grain, roots, &c.? How many gallons wine measure does it contain? How many bushels?

81. What is the use of the litre? How many quarts wine measure does it contain? How many quarts dry measure? Recite the table.

82. Give the rule for finding the contents of bins, cisterns, &c.

83. What unit of weight is commonly used in Philosophical experiments, by jewelers, &c.? What is its weight in Troy grains?

84. What is the use of the kilogram? How many pounds avoirdupois does it weigh?

85. What unit is commonly used in weighing heavy bodies, as iron, marble, R. R. freight, &c.? Recite the table.

86. Give the rule for finding the weight of water.

87. What method is used in the measurement of angles? Recite the table.

88. Recite the table of equivalents of linear measure. Recite the table of equivalents of square measure. Of cubic measure. Of weight. Of angular measure. Give rule for changing the common to the metric system.

89. *Give rule for* reducing the metric to the common system.

PROBLEMS IN COMPLEX ANALYSIS.

NOTE.—No solution will be given to the following problems, the object being, in compiling them, to give those pupils who desire, a chance to think. The answers may be easily verified.

101. A, B, and C can plow a field, working together, in 8 days; A and B can do it in 12 days; B and C in 15 days: in what time can each plow it alone?

102. A carpenter receives $3 per day, he pays $1 per day for his board; at the end of 50 days he finds he has $70 in money; how many days has he worked?

103. 80 quarts of maple sap will make 2 pounds of sugar; how much water must be added so that 10 quarts will make $\frac{1}{8}$ of a pound of sugar?

104. The time past noon equals $\frac{4}{5}$ of the time to midnight: what is the time?

105. Two-thirds of the time past noon equals $\frac{3}{4}$ of the time to midnight; what is the time?

106. What o'clock is it when the time past noon equals $\frac{2}{5}$ of the time past midnight?

107. There are two poles, the sum of their lengths is 80 ft.; the difference is 10 ft.; what is the length of each?

108. There is a fish, the head of which is $\frac{2}{5}$, and the tail $\frac{1}{3}$ of its length; the body is 8 inches; what is the length of the fish?

109. If every acre of meadow on a farm will keep 4 cows through the winter, and every acre of pasture will keep 2 cows through the summer, how many cows can be kept on a farm of 30 acres?

110. A farmer has 100 cows and sheep; the number of cows are to his sheep as 2 to 3; how many cows must he buy that the cows may be to his sheep as 2 to 1?

111. Two men buy 80 lbs. of pork for $3.20; A takes 50

pounds and pays $\frac{1}{2}$ cent more a pound than B; what is the price each pays per pound?

112. A man rowing against the current, while at work, propels himself at the rate of 8 miles per hour; but when idle the current carries him down at the rate of 2 miles per hour; how many hours will it take him to reach his home, 10 miles distant, if he is idle every alternate hour?

113. A man gives away $\frac{1}{5}$ of his money and $\frac{1}{5}$ of a dollar more; he then lends his neighbor $\frac{1}{4}$ of what he had left, and $\frac{1}{4}$ of a dollar more; he then pays a debt which takes $\frac{1}{3}$ of the remainder and $\frac{1}{3}$ of a dollar more; he afterwards buys a book with $\frac{1}{2}$ of what he then has and $\frac{1}{2}$ a dollar more; he finally gives the remainder, which is $3, to his wife; how much had he at first?

114. A boy found as much money as he had in his pocket, he then spent 10 cents for a loaf of bread; he then borrowed as much as he had left and then spent 10 cents for some tea; he then earned as much money as he then had, when he spent what he had left for some medicine, which was 10 cents; how much had he at first?

115. A man being asked his wife's age, said, "I have been married 15 years; then I was three times as old as my wife, but now I am only twice as old"; how old are each?

116. Fifty persons charter a boat for an excursion; 10 of the party decline going, for which reason the remainder are obliged to pay 25 cents each more than they otherwise would have done; what was paid for the use of the boat?

117. How far can a boy ride on a street car that goes at the rate of 12 miles an hour, if he can walk back at the rate of 3 miles per hour, and be gone from home only 4 hours?

118. Two boys together bought 8 melons, one paid for 3 and the other for 5 of them; they met a comrade with whom they agreed to eat the melons; the third boy gave the other two 24 cents for his share; how should they divide the money *between them?*

119. What is the length of a fish whose head is 9 inches, whose tail is as long as his head and $\frac{2}{3}$ of his body, and whose body is as long as the head and tail both?

120. A man has a mule, a horse and saddle; when the saddle is placed on the mule they together are worth 3 times as much as the horse; when the saddle is placed on the horse, they together are worth 4 times as much as the mule; what is the value of each if the saddle is worth $40?

121. John has 60 apples and Henry has 75 apples. John sells his apples at the rate of 2 for a cent, and Henry sells his at the rate of 3 for a cent; if they place them together and sell 5 for 2 cents, will they make or lose, how much, and why?

122. A boy buys a kite for 8 cents and sells it for 12 cents, he spends his 12 cents for a top, which he sells for 8 cents; what per cent. did he gain or lose in each transaction?

123. A grocer bought $60 worth of eggs at 15 cts. per dozen and sold them at 25 cts. per dozen; the next day he bought $60 worth of eggs at 25 cts. per dozen, and sold them at 12 cts. per dozen; what was his entire loss or gain?

124. What number is that which 2, 3, 4, 5, and 6 will divide with one for a remainder, and 7 will divide exactly?

125. A boy commenced trading, the first day he lost 14 cents, the next day he made $\frac{4}{5}$ as much as he had left; he then had $\frac{3}{4}$ as much as he had at first; how much had he at first?

126. John has 120 steps the start of William, and takes 5 steps while William takes 3; but 3 of William's steps are equal to 7 of John's steps; how many steps must William take before he can overtake John?

127. The hour and minute hands point in opposite directions at 6 o'clock; when will they be together?

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